

The background of the cover is a dark purple color with a complex technical drawing or blueprint overlaid in a lighter purple. The drawing features various geometric shapes, including circles, arcs, and lines, suggesting a mechanical or engineering theme. A large circular component with radial lines is prominent in the lower half of the image.

Introduction to

SPARK

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Introduction to SPARK

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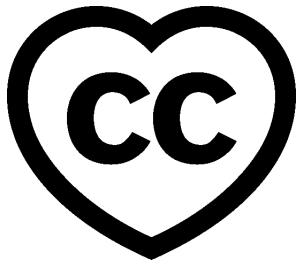
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This tutorial is an interactive introduction to the SPARK programming language and its formal verification tools. You will learn the difference between Ada and SPARK and how to use the various analysis tools that come with SPARK.

This document was prepared by Claire Dross and Yannick Moy.

Note: The code examples in this course use an 80-column limit, which is a typical limit for Ada code. Note that, on devices with a small screen size, some code examples might be difficult to read.

Note: Each code example from this book has an associated "code block metadata", which contains the name of the "project" and an MD5 hash value. This information is used to identify a single code example.

You can find all code examples in a zip file, which you can [download from the learn website](#)². The directory structure in the zip file is based on the code block metadata. For example, if you're searching for a code example with this metadata:

- Project: Courses.Intro_To_Ada.Imperative_Language.Greet
- MD5: cba89a34b87c9dfa71533d982d05e6ab

you will find it in this directory:

```
projects/Courses/Intro_To_Ada/Imperative_Language/Greet/  
cba89a34b87c9dfa71533d982d05e6ab/
```

In order to use this code example, just follow these steps:

1. Unpack the zip file;
 2. Go to target directory;
 3. Start GNAT Studio on this directory;
 4. Build (or compile) the project;
 5. Run the application (if a main procedure is available in the project).
-

¹ <http://creativecommons.org/licenses/by-sa/4.0>

² https://learn.adacore.com/zip/learning-ada_code.zip

SPARK OVERVIEW

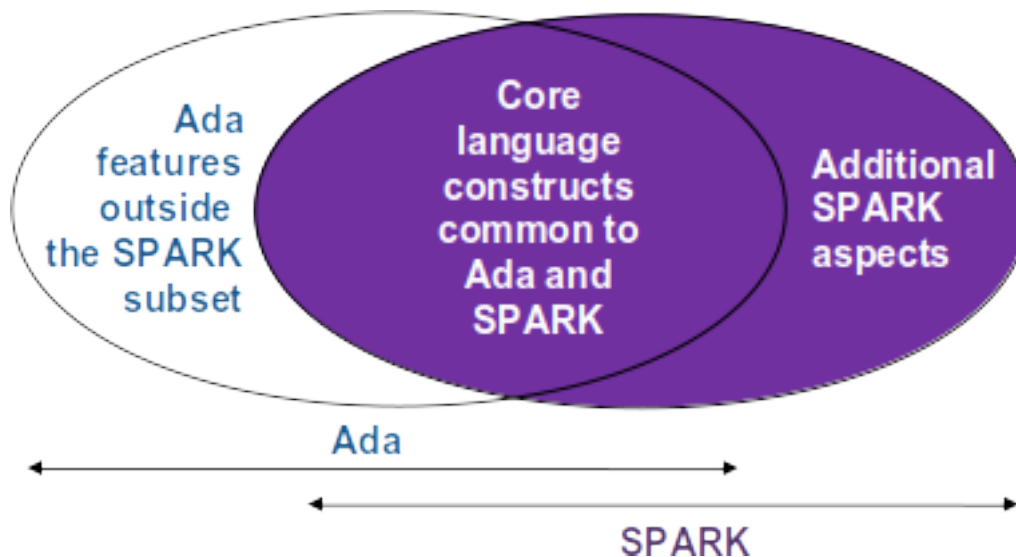
This tutorial is an introduction to the SPARK programming language and its formal verification tools. You need not know any specific programming language (although going over the Introduction to Ada course first may help) or have experience in formal verification.

1.1 What is it?

SPARK refers to two different things:

- a programming language targeted at functional specification and static verification, and
- a set of development and verification tools for that language.

The SPARK language is based on a subset of the Ada language. Ada is particularly well suited to formal verification since it was designed for critical software development. SPARK builds on that foundation.



Version 2012 of Ada introduced the use of *aspects*, which can be used for subprogram contracts, and version 2014 of SPARK added its own aspects to further aid static analysis.

1.2 What do the tools do?

We start by reviewing static verification of programs, which is verification of the source code performed without compiling or executing it. Verification uses tools that perform static analysis. These can take various forms. They include tools that check types and enforce visibility rules, such as the compiler, in addition to those that perform more complex reasoning, such as abstract interpretation, as done by a tool like [CodePeer](https://www.adacore.com/codepeer)³ from AdaCore. The tools that come with SPARK perform two different forms of static analysis:

- *flow analysis* is the fastest form of analysis. It checks initializations of variables and looks at data dependencies between inputs and outputs of subprograms. It can also find unused assignments and unmodified variables.
- *proof* checks for the absence of runtime errors as well as the conformance of the program with its specifications.

1.3 Key Tools

The tool for formal verification of the SPARK language is called *GNATprove*. It checks for conformance with the SPARK subset and performs flow analysis and proof of the source code. Several other tools support the SPARK language, including both the [GNAT compiler](https://www.adacore.com/gnatcompiler)⁴ and the [GNAT Studio integrated development environment](https://www.adacore.com/gnatstudio)⁵.

1.4 A trivial example

We start with a simple example of a subprogram in Ada that uses SPARK aspects to specify verifiable subprogram contracts. The subprogram, called `Increment`, adds 1 to the value of its parameter `X`:

Listing 1: `increment.ads`

```
1 procedure Increment
2   (X : in out Integer)
3 with
4   Global => null,
5   Depends => (X => X),
6   Pre => X < Integer'Last,
7   Post => X = X'Old + 1;
```

Listing 2: `increment.adb`

```
1 procedure Increment
2   (X : in out Integer)
3 is
4 begin
5   X := X + 1;
6 end Increment;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Overview.Trivial_Example
MD5: ce28b1facb44917b6cc208639c187064
```

³ <https://www.adacore.com/codepeer>

⁴ <https://www.adacore.com/gnatpro>

⁵ <https://www.adacore.com/gnatpro/toolsuite/gps>

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
increment.adb:5:10: info: overflow check proved
increment.ads:4:03: info: data dependencies proved
increment.ads:5:03: info: flow dependencies proved
increment.ads:7:14: info: postcondition proved
increment.ads:7:24: info: overflow check proved
```

The contracts are written using the Ada *aspect* feature and those shown specify several properties of this subprogram:

- The SPARK Global aspect says that Increment does not read or write any global variables.
- The SPARK Depend aspect is especially interesting for security: it says that the value of the parameter *X* after the call depends only on the (previous) value of *X*.
- The Pre and Post aspects of Ada specify functional properties of Increment:
 - Increment is only allowed to be called if the value of *X* prior to the call is less than `Integer'Last`. This ensures that the addition operation performed in the subprogram body doesn't overflow.
 - Increment does indeed perform an increment of *X*: the value of *X* after a call is one greater than its value before the call.

GNATprove can verify all of these contracts. In addition, it verifies that no error can be raised at runtime when executing Increment's body.

1.5 The Programming Language

It's important to understand why there are differences between the SPARK and Ada languages. The aim when designing the SPARK subset of Ada was to create the largest possible subset of Ada that was still amenable to simple specification and sound verification.

The most notable restrictions from Ada are related to exceptions and access types, both of which are known to considerably increase the amount of user-written annotations required for full support. Backwards goto statements and controlled types are also not supported since they introduce non-trivial control flow. The two remaining restrictions relate to side-effects in expressions and aliasing of names, which we now cover in more detail.

1.6 Limitations

1.6.1 No side-effects in expressions

The SPARK language doesn't allow side-effects in expressions. In other words, evaluating a SPARK expression must not update any object. This limitation is necessary to avoid unpredictable behavior that depends on order of evaluation, parameter passing mechanisms, or compiler optimizations. The expression for `Dummy` below is non-deterministic due to the order in which the two calls to `F` are evaluated. It's therefore not legal SPARK.

Listing 3: show_illegal_ada_code.adb

```
1 procedure Show_Illegal_Ada_Code is
2
3     function F (X : in out Integer) return Integer is
4         Tmp : constant Integer := X;
5     begin
6         X := X + 1;
7         return Tmp;
8     end F;
9
10    Dummy : Integer := 0;
11
12 begin
13     Dummy := F (Dummy) - F (Dummy); -- ??
14 end Show_Illegal_Ada_Code;
```

Code block metadata

Project: Courses.Intro_To_Spark.Overview.Illegal_Ada_Code
MD5: a5cbf1824526857da94791ac1790200c

Build output

```
show_illegal_ada_code.adb:13:28: error: value may be affected by call to "F"
↳because order of evaluation is arbitrary
gprbuild: *** compilation phase failed
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
show_illegal_ada_code.adb:13:28: error: value may be affected by call to "F"
↳because order of evaluation is arbitrary
gnatprove: error during generation of Global contracts
```

In fact, the code above is not even legal Ada, so the same error is generated by the GNAT compiler. But SPARK goes further and GNATprove also produces an error for the following equivalent code that is accepted by the Ada compiler:

Listing 4: show_illegal_spark_code.adb

```
1 procedure Show_Illegal_SPARK_Code is
2
3     Dummy : Integer := 0;
4
5     function F return Integer is
6         Tmp : constant Integer := Dummy;
7     begin
8         Dummy := Dummy + 1;
9         return Tmp;
10    end F;
11
12 begin
13     Dummy := F - F; -- ??
14 end Show_Illegal_SPARK_Code;
```

Code block metadata

Project: Courses.Intro_To_Spark.Overview.Illegal_SPARK_Code
MD5: e747edb6ee147adb7fba97c9e7c8d5ef

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_illegal_spark_code.adb:5:13: error: function with output global "Dummy" is
↳not allowed in SPARK
gnatprove: error during analysis of data and information flow
```

The SPARK languages enforces the lack of side-effects in expressions by forbidding side-effects in functions, which include modifications to either parameters or global variables. As a consequence, SPARK forbids functions with **out** or **in out** parameters in addition to functions modifying a global variable. Function F below is illegal in SPARK, while Function Incr might be legal if it doesn't modify any global variables and function Incr_And_Log might be illegal if it modifies global variables to perform logging.

```
function F (X : in out Integer) return Integer;    -- Illegal
function Incr (X : Integer) return Integer;       -- OK?
function Incr_And_Log (X : Integer) return Integer; -- OK?
```

In most cases, you can easily replace these functions by procedures with an **out** parameter that returns the computed value.

When it has access to function bodies, GNATprove verifies that those functions are indeed free from side-effects. Here for example, the two functions Incr and Incr_And_Log have the same signature, but only Incr is legal in SPARK. Incr_And_Log isn't: it attempts to update the global variable Call_Count.

Listing 5: side_effects.ads

```
1 package Side_Effects is
2
3     function Incr (X : Integer) return Integer;    -- OK?
4
5     function Incr_And_Log (X : Integer) return Integer; -- OK?
6
7 end Side_Effects;
```

Listing 6: side_effects.adb

```
1 package body Side_Effects is
2
3     function Incr (X : Integer) return Integer
4     is (X + 1); -- OK
5
6     Call_Count : Natural := 0;
7
8     function Incr_And_Log (X : Integer) return Integer is
9     begin
10        Call_Count := Call_Count + 1; -- Illegal
11        return X + 1;
12    end Incr_And_Log;
13
14 end Side_Effects;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Overview.Side_Effects
MD5: 1b555e4b7bb519eea4df718a9356a2ed
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
side_effects.ads:5:13: error: function with output global "Call_Count" is not
↳ allowed in SPARK
gnatprove: error during analysis of data and information flow
```

1.6.2 No aliasing of names

Another restriction imposed by the SPARK subset concerns [aliasing](#)⁶. We say that two names are *aliased* if they refer to the same object. There are two reasons why aliasing is forbidden in SPARK:

- It makes verification more difficult because it requires taking into account the fact that modifications to variables with different names may actually update the same object.
- Results may seem unexpected from a user point of view. The results of a subprogram call may depend on compiler-specific attributes, such as parameter passing mechanisms, when its parameters are aliased.

Aliasing can occur as part of the parameter passing that occurs in a subprogram call. Functions have no side-effects in SPARK, so aliasing of parameters in function calls isn't problematic; we need only consider procedure calls. When a procedure is called, SPARK verifies that no **out** or **in out** parameter is aliased with either another parameter of the procedure or a global variable modified in the procedure's body.

Procedure `Move_To_Total` is an example where the possibility of aliasing wasn't taken into account by the programmer:

Listing 7: no_aliasing.adb

```
1  procedure No_Aliasing is
2
3     Total : Natural := 0;
4
5     procedure Move_To_Total (Source : in out Natural)
6         with Post => Total = Total'Old + Source'Old and Source = 0
7     is
8     begin
9         Total := Total + Source;
10        Source := 0;
11    end Move_To_Total;
12
13    X : Natural := 3;
14
15    begin
16        Move_To_Total (X);           -- OK
17        pragma Assert (Total = 3);  -- OK
18        Move_To_Total (Total);      -- flow analysis error
19        pragma Assert (Total = 6);  -- runtime error
20    end No_Aliasing;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Overview.Aliasing
MD5: 91038ef030fe27e3b000ab3db9c134ad
```

Prover output

⁶ [https://en.wikipedia.org/wiki/Aliasing_\(computing\)](https://en.wikipedia.org/wiki/Aliasing_(computing))

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
no_aliasing.adb:18:19: high: formal parameter "Source" and global "Total" are
↳ aliased (SPARK RM 6.4.2)
gnatprove: unproved check messages considered as errors
```

Runtime output

```
raised ADA.ASSERTIONS.ASSERTION_ERROR : no_aliasing.adb:19
```

`Move_To_Total` adds the value of its input parameter `Source` to the global variable `Total` and then resets `Source` to 0. The programmer has clearly not taken into account the possibility of an aliasing between `Total` and `Source`. (This sort of error is quite common.)

This procedure itself is valid SPARK. When doing verification, GNATprove assumes, like the programmer did, that there's no aliasing between `Total` and `Source`. To ensure this assumption is valid, GNATprove checks for possible aliasing on every call to `Move_To_Total`. Its final call in procedure `No_Aliasing` violates this assumption, which produces both a message from GNATprove and a runtime error (an assertion violation corresponding to the expected change in `Total` from calling `Move_To_Total`). Note that the postcondition of `Move_To_Total` is not violated on this second call since integer parameters are passed by copy and the postcondition is checked before the copy-back from the formal parameters to the actual arguments.

Aliasing can also occur as a result of using access types ([pointers](#)⁷ in Ada). These are restricted in SPARK so that only benign aliasing is allowed, when both names are only used to read the data. In particular, assignment between access objects operates a transfer of ownership, where the source object loses its permission to read or write the underlying allocated memory.

Procedure `Ownership_Transfer` is an example of code that is legal in Ada but rejected in SPARK due to aliasing:

Listing 8: `ownership_transfer.adb`

```
1 procedure Ownership_Transfer is
2   type Int_Ptr is access Integer;
3   X      : Int_Ptr;
4   Y      : Int_Ptr;
5   Dummy : Integer;
6 begin
7   X      := new Integer'(1);
8   X.all := X.all + 1;
9   Y      := X;
10  Y.all := Y.all + 1;
11  X.all := X.all + 1; -- illegal
12  X.all := 1;        -- illegal
13  Dummy := X.all;   -- illegal
14 end Ownership_Transfer;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Overview.Ownership_Transfer
MD5: 951fe1c930d43a5009e607994ae0dd03
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
```

(continues on next page)

⁷ [https://en.wikipedia.org/wiki/Pointer_\(computer_programming\)](https://en.wikipedia.org/wiki/Pointer_(computer_programming))

(continued from previous page)

```
ownership_transfer.adb:11:06: error: dereference from "X" is not writable
ownership_transfer.adb:11:06: error: object was moved at line 9
ownership_transfer.adb:11:15: error: dereference from "X" is not readable
ownership_transfer.adb:11:15: error: object was moved at line 9
ownership_transfer.adb:12:06: error: dereference from "X" is not writable
ownership_transfer.adb:12:06: error: object was moved at line 9
ownership_transfer.adb:13:15: error: dereference from "X" is not readable
ownership_transfer.adb:13:15: error: object was moved at line 9
gnatprove: error during analysis of data and information flow
```

After the assignment of X to Y, variable X cannot be used anymore to read or write the underlying allocated memory.

Note: For more details on these limitations, see the [SPARK User's Guide](#)⁸.

1.7 Designating SPARK Code

Since the SPARK language is restricted to only allow easily specifiable and verifiable constructs, there are times when you can't or don't want to abide by these limitations over your entire code base. Therefore, the SPARK tools only check conformance to the SPARK subset on code which you identify as being in SPARK.

You do this by using an aspect named `SPARK_Mode`. If you don't explicitly specify otherwise, `SPARK_Mode` is *Off*, meaning you can use the complete set of Ada features in that code and that it should not be analyzed by GNATprove. You can change this default either selectively (on some units or subprograms or packages inside units) or globally (using a configuration pragma, which is what we're doing in this tutorial). To allow simple reuse of existing Ada libraries, entities declared in imported units with no explicit `SPARK_Mode` can still be used from SPARK code. The tool only checks for SPARK conformance on the declaration of those entities which are actually used within the SPARK code.

Here's a common case of using the `SPARK_Mode` aspect:

```
package P
  with SPARK_Mode => On
is
  -- package spec is IN SPARK, so can be used by SPARK clients
end P;

package body P
  with SPARK_Mode => Off
is
  -- body is NOT IN SPARK, so is ignored by GNATprove
end P;
```

The package P only defines entities whose specifications are in the SPARK subset. However, it wants to use all Ada features in its body. Therefore the body should not be analyzed and has its `SPARK_Mode` aspect set to *Off*.

You can specify `SPARK_Mode` in a fine-grained manner on a per-unit basis. An Ada package has four different components: the visible and private parts of its specification and the declarative and statement parts of its body. You can specify `SPARK_Mode` as being either *On* or *Off* on any of those parts. Likewise, a subprogram has two parts: its specification and its body.

⁸ https://docs.adacore.com/live/wave/spark2014/html/spark2014_ug/en/source/language_restrictions.html#language-restrictions

A general rule in SPARK is that once `SPARK_Mode` has been set to *Off*, it can never be switched *On* again in the same part of a package or subprogram. This prevents setting `SPARK_Mode` to *On* for subunits of a unit with `SPARK_Mode Off` and switching back to `SPARK_Mode On` for a part of a given unit where it was set to *Off* in a previous part.

Note: For more details on the use of `SPARK_Mode`, see the [SPARK User's Guide⁹](#).

1.8 Code Examples / Pitfalls

1.8.1 Example #1

Here's a package defining an abstract stack type (defined as a private type in SPARK) of `Element` objects along with some subprograms providing the usual functionalities of stacks. It's marked as being in the SPARK subset.

Listing 9: `stack_package.ads`

```

1 package Stack_Package
2   with SPARK_Mode => On
3 is
4   type Element is new Natural;
5   type Stack is private;
6
7   function Empty return Stack;
8   procedure Push (S : in out Stack; E : Element);
9   function Pop (S : in out Stack) return Element;
10
11 private
12   type Stack is record
13     Top : Integer;
14     -- ...
15   end record;
16
17 end Stack_Package;
```

Code block metadata

Project: Courses.Intro_To_Spark.Overview.Example_01
MD5: 2b15e13e850435fb93406054d70b51c6

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
stack_package.ads:9:13: error: function with "in out" parameter is not allowed in_
↳ SPARK
stack_package.ads:9:13: error: violation of aspect SPARK_Mode at line 2
gnatprove: error during analysis of data and information flow
```

Side-effects in expressions are not allowed in SPARK. Therefore, `Pop` is not allowed to modify its parameter `S`.

⁹ https://docs.adacore.com/live/wave/spark2014/html/spark2014_ug/en/spark_mode.html

1.8.2 Example #2

Let's turn to an abstract state machine version of a stack, where the unit provides a single instance of a stack. The content of the stack (global variables `Content` and `Top`) is not directly visible to clients. In this stripped-down version, only the function `Pop` is available to clients. The package spec and body are marked as being in the SPARK subset.

Listing 10: `global_stack.ads`

```
1 package Global_Stack
2   with SPARK_Mode => On
3 is
4   type Element is new Integer;
5
6   function Pop return Element;
7
8 end Global_Stack;
```

Listing 11: `global_stack.adb`

```
1 package body Global_Stack
2   with SPARK_Mode => On
3 is
4   Max : constant Natural := 100;
5   type Element_Array is array (1 .. Max) of Element;
6
7   Content : Element_Array;
8   Top     : Natural;
9
10  function Pop return Element is
11    E : constant Element := Content (Top);
12  begin
13    Top := Top - 1;
14    return E;
15  end Pop;
16
17 end Global_Stack;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Overview.Example_02
MD5: 8c4eb564643eef48264b5e43a6f580b9
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
global_stack.adb:7:04: warning: variable "Content" is read but never assigned [-
↳gnatwv]
global_stack.ads:6:13: error: function with output global "Top" is not allowed in
↳SPARK
gnatprove: error during analysis of data and information flow
```

As above, functions should be free from side-effects. Here, `Pop` updates the global variable `Top`, which is not allowed in SPARK.

1.8.3 Example #3

We now consider two procedures: `Permute` and `Swap`. `Permute` applies a circular permutation to the value of its three parameters. `Swap` then uses `Permute` to swap the value of `X` and `Y`.

Listing 12: p.ads

```

1 package P
2   with SPARK_Mode => On
3 is
4   procedure Permute (X, Y, Z : in out Positive);
5   procedure Swap (X, Y : in out Positive);
6 end P;
```

Listing 13: p.adb

```

1 package body P
2   with SPARK_Mode => On
3 is
4   procedure Permute (X, Y, Z : in out Positive) is
5     Tmp : constant Positive := X;
6   begin
7     X := Y;
8     Y := Z;
9     Z := Tmp;
10  end Permute;
11
12  procedure Swap (X, Y : in out Positive) is
13  begin
14    Permute (X, Y, Y);
15  end Swap;
16 end P;
```

Listing 14: test_swap.adb

```

1 with P; use P;
2
3 procedure Test_Swap
4   with SPARK_Mode => On
5 is
6   A : Integer := 1;
7   B : Integer := 2;
8 begin
9   Swap (A, B);
10 end Test_Swap;
```

Code block metadata

Project: Courses.Intro_To_Spark.Overview.Example_03
MD5: 0868a806061d86af4d2a03b1e7dc83c2

Build output

p.adb:14:19: error: writable actual for "Y" overlaps with actual for "Z"
gprbuild: *** compilation phase failed

Prover output

Phase 1 of 2: generation of Global contracts ...
p.adb:14:19: error: writable actual for "Y" overlaps with actual for "Z"
gnatprove: error during generation of Global contracts

Here, the values for parameters Y and Z are aliased in the call to `Permute`, which is not allowed in SPARK. In fact, in this particular case, this is even a violation of Ada rules so the same error is issued by the Ada compiler.

In this example, we see the reason why aliasing is not allowed in SPARK: since Y and Z are **Positive**, they are passed by copy and the result of the call to `Permute` depends on the order in which they're copied back after the call.

1.8.4 Example #4

Here, the `Swap` procedure is used to swap the value of the two record components of R.

Listing 15: p.ads

```
1 package P
2   with SPARK_Mode => On
3 is
4   type Rec is record
5     F1 : Positive;
6     F2 : Positive;
7   end record;
8
9   procedure Swap_Fields (R : in out Rec);
10  procedure Swap (X, Y : in out Positive);
11 end P;
```

Listing 16: p.adb

```
1 package body P
2   with SPARK_Mode => On
3 is
4   procedure Swap (X, Y : in out Positive) is
5     Tmp : constant Positive := X;
6   begin
7     X := Y;
8     Y := Tmp;
9   end Swap;
10
11  procedure Swap_Fields (R : in out Rec) is
12  begin
13    Swap (R.F1, R.F2);
14  end Swap_Fields;
15
16 end P;
```

Code block metadata

Project: Courses.Intro_To_Spark.Overview.Example_04
MD5: ae4d3ebe8dd1a8f67f35cedffdea2ac9

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...

This code is correct. The call to `Swap` is safe: two different components of the same record can't refer to the same object.

1.8.5 Example #5

Here's a slight modification of the previous example using an array instead of a record: Swap_Indexes calls Swap on values stored in the array A.

Listing 17: p.ads

```

1 package P
2   with SPARK_Mode => On
3 is
4   type P_Array is array (Natural range <>) of Positive;
5
6   procedure Swap_Indexes (A : in out P_Array; I, J : Natural);
7   procedure Swap (X, Y : in out Positive);
8 end P;
```

Listing 18: p.adb

```

1 package body P
2   with SPARK_Mode => On
3 is
4   procedure Swap (X, Y : in out Positive) is
5     Tmp : constant Positive := X;
6   begin
7     X := Y;
8     Y := Tmp;
9   end Swap;
10
11  procedure Swap_Indexes (A : in out P_Array; I, J : Natural) is
12  begin
13    Swap (A (I), A (J));
14  end Swap_Indexes;
15
16 end P;
```

Code block metadata

Project: Courses.Intro_To_Spark.Overview.Example_05
MD5: 62a95179572e36443995ff54a2d5ef08

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
p.adb:13:13: medium: formal parameters "X" and "Y" might be aliased (SPARK RM 6.4.
↪2)
gnatprove: unproved check messages considered as errors
```

GNATprove detects a possible case of aliasing. Unlike the previous example, it has no way of knowing that the two elements A (I) and A (J) are actually distinct when we call Swap. GNATprove issues a check message here instead of an error, giving you the possibility of justifying the message after review (meaning that you've verified manually that this can't, in fact, occur).

1.8.6 Example #6

We now consider a package declaring a type `Dictionary`, an array containing a word per letter. The procedure `Store` allows us to insert a word at the correct index in a dictionary.

Listing 19: p.ads

```
1 with Ada.Finalization;
2
3 package P
4   with SPARK_Mode => On
5 is
6   subtype Letter is Character range 'a' .. 'z';
7   type String_Access is new Ada.Finalization.Controlled with record
8     Ptr : access String;
9   end record;
10  type Dictionary is array (Letter) of String_Access;
11
12  procedure Store (D : in out Dictionary; W : String);
13 end P;
```

Listing 20: p.adb

```
1 package body P
2   with SPARK_Mode => On
3 is
4   procedure Store (D : in out Dictionary; W : String) is
5     First_Letter : constant Letter := W (W'First);
6   begin
7     D (First_Letter).Ptr := new String'(W);
8   end Store;
9 end P;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Overview.Example_06
MD5: 9175bcd1474e2143462b860c01d8602e
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
p.adb:7:07: error: "String_Access" is not allowed in SPARK (due to controlled_
↳types)
p.adb:7:07: error: violation of aspect SPARK_Mode at line 2
p.adb:7:31: error: borrow or observe of an expression which is not part of stand-
↳alone object or parameter is not allowed in SPARK (SPARK RM 3.10(3))
p.adb:7:31: error: violation of aspect SPARK_Mode at line 2
p.ads:7:09: error: "Controlled" is not allowed in SPARK (due to controlled types)
p.ads:7:09: error: violation of aspect SPARK_Mode at line 4
p.ads:10:04: error: "String_Access" is not allowed in SPARK (due to controlled_
↳types)
p.ads:10:04: error: violation of aspect SPARK_Mode at line 4
gnatprove: error during analysis of data and information flow
```

This code is not correct: controlled types are not part of the SPARK subset. The solution here is to use `SPARK_Mode` to separate the definition of `String_Access` from the rest of the code in a fine grained manner.

1.8.7 Example #7

Here's a new version of the previous example, which we've modified to hide the controlled type inside the private part of package P, using pragma SPARK_Mode (Off) at the start of the private part.

Listing 21: p.ads

```

1 with Ada.Finalization;
2
3 package P
4   with SPARK_Mode => On
5 is
6   subtype Letter is Character range 'a' .. 'z';
7   type String_Access is private;
8   type Dictionary is array (Letter) of String_Access;
9
10  function New_String_Access (W : String) return String_Access;
11
12  procedure Store (D : in out Dictionary; W : String);
13
14 private
15  pragma SPARK_Mode (Off);
16
17  type String_Access is new Ada.Finalization.Controlled with record
18    Ptr : access String;
19  end record;
20
21  function New_String_Access (W : String) return String_Access is
22    (Ada.Finalization.Controlled with Ptr => new String'(W));
23 end P;
```

Code block metadata

Project: Courses.Intro_To_Spark.Overview.Example_07
 MD5: cb04206c9734eb95f6444757d005dae2

Prover output

Phase 1 of 2: generation of Global contracts ...
 Phase 2 of 2: analysis of data and information flow ...

Since the controlled type is defined and used inside of a part of the code ignored by GNAT-Prove, this code is correct.

1.8.8 Example #8

Let's put together the new spec for package P with the body of P seen previously.

Listing 22: p.ads

```

1 with Ada.Finalization;
2
3 package P
4   with SPARK_Mode => On
5 is
6   subtype Letter is Character range 'a' .. 'z';
7   type String_Access is private;
8   type Dictionary is array (Letter) of String_Access;
9
```

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```
10  function New_String_Access (W : String) return String_Access;
11
12  procedure Store (D : in out Dictionary; W : String);
13
14  private
15    pragma SPARK_Mode (Off);
16
17    type String_Access is new Ada.Finalization.Controlled with record
18      Ptr : access String;
19    end record;
20
21    function New_String_Access (W : String) return String_Access is
22      (Ada.Finalization.Controlled with Ptr => new String'(W));
23  end P;
```

Listing 23: p.adb

```
1  package body P
2    with SPARK_Mode => On
3  is
4    procedure Store (D : in out Dictionary; W : String) is
5      First_Letter : constant Letter := W (W'First);
6    begin
7      D (First_Letter) := New_String_Access (W);
8    end Store;
9  end P;
```

Code block metadata

Project: Courses.Intro_To_Spark.Overview.Example_08
MD5: dacb2d50d0ddc6c620ee9945cb819369

Prover output

```
Phase 1 of 2: generation of Global contracts ...
p.adb:1:01: error: incorrect application of SPARK_Mode at /vagrant/frontend/dist/
↳test_output/projects/Courses/Intro_To_Spark/Overview/Example_08/
↳dacb2d50d0ddc6c620ee9945cb819369/main_spark.adc:12
p.adb:1:01: error: value Off was set for SPARK_Mode on "P" at p.ads:15
p.adb:2:08: error: incorrect use of SPARK_Mode
p.adb:2:08: error: value Off was set for SPARK_Mode on "P" at p.ads:15
gnatprove: error during generation of Global contracts
```

The body of Store doesn't actually use any construct that's not in the SPARK subset, but we nevertheless can't set SPARK_Mode to On for P's body because it has visibility to P's private part, which is not in SPARK, even if we don't use it.

1.8.9 Example #9

Next, we moved the declaration and the body of the procedure Store to another package named Q.

Listing 24: p.ads

```
1  with Ada.Finalization;
2
3  package P
4    with SPARK_Mode => On
5  is
```

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```

6  subtype Letter is Character range 'a' .. 'z';
7  type String_Access is private;
8  type Dictionary is array (Letter) of String_Access;
9
10 function New_String_Access (W : String) return String_Access;
11
12 private
13   pragma SPARK_Mode (Off);
14
15   type String_Access is new Ada.Finalization.Controlled with record
16     Ptr : access String;
17   end record;
18
19   function New_String_Access (W : String) return String_Access is
20     (Ada.Finalization.Controlled with Ptr => new String'(W));
21 end P;
```

Listing 25: q.ads

```

1  with P; use P;
2  package Q
3    with SPARK_Mode => On
4  is
5    procedure Store (D : in out Dictionary; W : String);
6  end Q;
```

Listing 26: q.adb

```

1  package body Q
2    with SPARK_Mode => On
3  is
4    procedure Store (D : in out Dictionary; W : String) is
5      First_Letter : constant Letter := W (W'First);
6    begin
7      D (First_Letter) := New_String_Access (W);
8    end Store;
9  end Q;
```

Code block metadata

Project: Courses.Intro_To_Spark.Overview.Example_09
MD5: b397e82987c100de5a53ede16fbef37f

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...

And now everything is fine: we've managed to retain the use of the controlled type while having most of our code in the SPARK subset so GNATprove is able to analyze it.

1.8.10 Example #10

Our final example is a package with two functions to search for the value 0 inside an array A. The first raises an exception if 0 isn't found in A while the other simply returns 0 in that case.

Listing 27: p.ads

```
1 package P
2   with SPARK_Mode => On
3 is
4   type N_Array is array (Positive range <>) of Natural;
5   Not_Found : exception;
6
7   function Search_Zero_P (A : N_Array) return Positive;
8
9   function Search_Zero_N (A : N_Array) return Natural;
10 end P;
```

Listing 28: p.adb

```
1 package body P
2   with SPARK_Mode => On
3 is
4   function Search_Zero_P (A : N_Array) return Positive is
5   begin
6     for I in A'Range loop
7       if A (I) = 0 then
8         return I;
9       end if;
10    end loop;
11    raise Not_Found;
12 end Search_Zero_P;
13
14 function Search_Zero_N (A : N_Array) return Natural
15 with SPARK_Mode => Off is
16 begin
17   return Search_Zero_P (A);
18 exception
19   when Not_Found => return 0;
20 end Search_Zero_N;
21 end P;
```

Code block metadata

Project: Courses.Intro_To_Spark.Overview.Example_10
MD5: 4b9656698ab1d42cebc72817f8a00637

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
p.adb:11:07: medium: exception might be raised
gnatprove: unproved check messages considered as errors
```

This code is perfectly correct, despite the use of exception handling, because we've carefully isolated this non-SPARK feature in a function body marked with a SPARK_Mode of Off so it's ignored by GNATprove. However, GNATprove tries to show that Not_Found is never raised in Search_Zero_P, producing a message about a possible exception being raised. Looking at Search_Zero_N, it's indeed likely that an exception is meant to be raised in some cases, which means you need to verify that Not_Found is only raised when appropriate using other methods such as peer review or testing.

FLOW ANALYSIS

In this section we present the flow analysis capability provided by the GNATprove tool, a critical tool for using SPARK.

2.1 What does flow analysis do?

Flow analysis concentrates primarily on variables. It models how information flows through them during a subprogram's execution, connecting the final values of variables to their initial values. It analyzes global variables declared at library level, local variables, and formal parameters of subprograms.

Nesting of subprograms creates what we call *scope variables*: variables declared locally to an enclosing unit. From the perspective of a nested subprogram, scope variables look very much like global variables

Flow analysis is usually fast, roughly as fast as compilation. It detects various types of errors and finds violations of some SPARK legality rules, such as the absence of aliasing and freedom of expressions from side-effects. We discussed these rules in the [SPARK Overview](#) (page 3).

Flow analysis is *sound*: if it doesn't detect any errors of a type it's supposed to detect, we know for sure there are no such errors.

2.2 Errors Detected

2.2.1 Uninitialized Variables

We now present each class of errors detected by flow analysis. The first is the reading of an uninitialized variable. This is nearly always an error: it introduces non-determinism and breaks the type system because the value of an uninitialized variable may be outside the range of its subtype. For these reasons, SPARK requires every variable to be initialized before being read.

Flow analysis is responsible for ensuring that SPARK code always fulfills this requirement. For example, in the function `Max_Array` shown below, we've neglected to initialize the value of `Max` prior to entering the loop. As a consequence, the value read by the condition of the `if` statement may be uninitialized. Flow analysis detects and reports this error.

Listing 1: `show_uninitialized.ads`

```
1 package Show_Uninitialized is
2
3   type Array_Of_Naturals is array (Integer range <>) of Natural;
```

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```
4
5     function Max_Array (A : Array_Of_Naturals) return Natural;
6
7 end Show_Uninitialized;
```

Listing 2: show_uninitialized.adb

```
1 package body Show_Uninitialized is
2
3     function Max_Array (A : Array_Of_Naturals) return Natural is
4         Max : Natural;
5     begin
6         for I in A'Range loop
7             if A (I) > Max then -- Here Max may not be initialized
8                 Max := A (I);
9             end if;
10        end loop;
11        return Max;
12    end Max_Array;
13
14 end Show_Uninitialized;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Flow_Analysis.Uninitialized
MD5: 82fe32cbe33e25bac5466f86ee2e03c4
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_uninitialized.adb:7:21: warning: "Max" may be referenced before it has a
↳value [enabled by default]
show_uninitialized.adb:7:21: medium: "Max" might not be initialized
show_uninitialized.adb:11:14: medium: "Max" might not be initialized
gnatprove: unproved check messages considered as errors
```

Note: For more details on how flow analysis verifies data initialization, see the [SPARK User's Guide](#)¹⁰.

2.2.2 Ineffective Statements

Ineffective statements are different than dead code: they're executed, and often even modify the value of variables, but have no effect on any of the subprogram's visible outputs: parameters, global variables or the function result. Ineffective statements should be avoided because they make the code less readable and more difficult to maintain.

More importantly, they're often caused by errors in the program: the statement may have been written for some purpose, but isn't accomplishing that purpose. These kinds of errors can be difficult to detect in other ways.

For example, the subprograms `Swap1` and `Swap2` shown below don't properly swap their two parameters `X` and `Y`. This error caused a statement to be ineffective. That ineffective statement is not an error in itself, but flow analysis produces a warning since it can be indicative of an error, as it is here.

¹⁰ https://docs.adacore.com/live/wave/spark2014/html/spark2014_ug/en/source/language_restrictions.html#data-initialization-policy

Listing 3: show_ineffective_statements.ads

```

1 package Show_Ineffective_Statements is
2
3   type T is new Integer;
4
5   procedure Swap1 (X, Y : in out T);
6   procedure Swap2 (X, Y : in out T);
7
8 end Show_Ineffective_Statements;
```

Listing 4: show_ineffective_statements.adb

```

1 package body Show_Ineffective_Statements is
2
3   procedure Swap1 (X, Y : in out T) is
4     Tmp : T;
5   begin
6     Tmp := X; -- This statement is ineffective
7     X := Y;
8     Y := X;
9   end Swap1;
10
11  Tmp : T := 0;
12
13  procedure Swap2 (X, Y : in out T) is
14    Temp : T := X; -- This variable is unused
15  begin
16    X := Y;
17    Y := Temp;
18  end Swap2;
19
20 end Show_Ineffective_Statements;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Flow_Analysis.Ineffective_Statements
MD5: 473a9215e9e98bd25147998d43847a12
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_ineffective_statements.adb:6:11: warning: unused assignment
show_ineffective_statements.adb:14:07: warning: initialization of "Temp" has no
↳effect
show_ineffective_statements.ads:5:21: warning: unused initial value of "X"
show_ineffective_statements.ads:6:21: warning: unused initial value of "X"
```

So far, we've seen examples where flow analysis warns about ineffective statements and unused variables.

2.2.3 Incorrect Parameter Mode

Parameter modes are an important part of documenting the usage of a subprogram and affect the code generated for that subprogram. Flow analysis checks that each specified parameter mode corresponds to the usage of that parameter in the subprogram's body. It checks that an **in** parameter is never modified, either directly or through a subprogram call, checks that the initial value of an **out** parameter is never read in the subprogram (since it may not be defined on subprogram entry), and warns when an **in out** parameter isn't modified or when its initial value isn't used. All of these may be signs of an error.

We see an example below. The subprogram `Swap` is incorrect and GNATprove warns about an input which isn't read:

Listing 5: `show_incorrect_param_mode.ads`

```

1 package Show_Incorrect_Param_Mode is
2
3   type T is new Integer;
4
5   procedure Swap (X, Y : in out T);
6
7 end Show_Incorrect_Param_Mode;
```

Listing 6: `show_incorrect_param_mode.adb`

```

1 package body Show_Incorrect_Param_Mode is
2
3   procedure Swap (X, Y : in out T) is
4     Tmp : T := X;
5   begin
6     Y := X;  -- The initial value of Y is not used
7     X := Tmp; -- Y is computed to be an out parameter
8   end Swap;
9
10 end Show_Incorrect_Param_Mode;
```

Code block metadata

Project: `Courses.Intro_To_Spark.Flow_Analysis.Incorrect_Param_Mode`
MD5: `1e33dbf461daab0daed01c83025232fc`

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_incorrect_param_mode.ads:5:23: warning: unused initial value of "Y"
```

In SPARK, unlike Ada, you should declare an **out** parameter to be **in out** if it's not modified on every path, in which case its value may depend on its initial value. SPARK is stricter than Ada to allow more static detection of errors. This table summarizes SPARK's valid parameter modes as a function of whether reads and writes are done to the parameter.

Initial value read	Written on some path	Written on every path	Parameter mode
X			in
X	X		in out
X		X	in out
	X		in out
		X	out

2.3 Additional Verifications

2.3.1 Global Contracts

So far, none of the verifications we've seen require you to write any additional annotations. However, flow analysis also checks flow annotations that you write. In SPARK, you can specify the set of global and scoped variables accessed or modified by a subprogram. You do this using a contract named `Global`.

When you specify a `Global` contract for a subprogram, flow analysis checks that it's both correct and complete, meaning that no variables other than those stated in the contract are accessed or modified, either directly or through a subprogram call, and that all those listed are accessed or modified. For example, we may want to specify that the function `Get_Value_Of_X` reads the value of the global variable `X` and doesn't access any other global variable. If we do this through a comment, as is usually done in other languages, GNATprove can't verify that the code complies with this specification:

```
package Show_Global_Contracts is
  X : Natural := 0;

  function Get_Value_Of_X return Natural;
  -- Get_Value_Of_X reads the value of the global variable X

end Show_Global_Contracts;
```

You write global contracts as part of the subprogram specification. In addition to their value in flow analysis, they also provide useful information to users of a subprogram. The value you specify for the `Global` aspect is an aggregate-like list of global variable names, grouped together according to their mode.

In the example below, the procedure `Set_X_To_Y_Plus_Z` reads both `Y` and `Z`. We indicate this by specifying them as the value for `Input`. It also writes `X`, which we specify using `Output`. Since `Set_X_To_X_Plus_Y` both writes `X` and reads its initial value, `X`'s mode is `In_Out`. Like parameters, if no mode is specified in a `Global` aspect, the default is `Input`. We see this in the case of the declaration of `Get_Value_Of_X`. Finally, if a subprogram, such as `Incr_Parameter_X`, doesn't reference any global variables, you set the value of the global contract to `null`.

Listing 7: show_global_contracts.ads

```
1 package Show_Global_Contracts is
2
3   X, Y, Z : Natural := 0;
4
5   procedure Set_X_To_Y_Plus_Z with
6     Global => (Input => (Y, Z), -- reads values of Y and Z
7               Output => X);    -- modifies value of X
8
9   procedure Set_X_To_X_Plus_Y with
10    Global => (Input => Y, -- reads value of Y
11              In_Out => X); -- modifies value of X and
12                          -- also reads its initial value
13
14   function Get_Value_Of_X return Natural with
15     Global => X; -- reads the value of the global variable X
16
17   procedure Incr_Parameter_X (X : in out Natural) with
18     Global => null; -- do not reference any global variable
```

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```
19
20 end Show_Global_Contracts;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Flow_Analysis.Global_Contracts
MD5: 2cbf90f2d27b6b0043a2e29449e79df9
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
```

Note: For more details on global contracts, see the [SPARK User's Guide](#)¹¹.

2.3.2 Depends Contracts

You may also supply a Depends contract for a subprogram to specify dependencies between its inputs and outputs. These dependencies include not only global variables but also parameters and the function's result. When you supply a Depends contract for a subprogram, flow analysis checks that it's correct and complete, that is, for each dependency you list, the variable depends on those listed and on no others.

For example, you may want to say that the new value of each parameter of `Swap`, shown below, depends only on the initial value of the other parameter and that the value of `X` after the return of `Set_X_To_Zero` doesn't depend on any global variables. If you indicate this through a comment, as you often do in other languages, GNATprove can't verify that this is actually the case.

```
package Show_Depends_Contracts is

  type T is new Integer;

  procedure Swap (X, Y : in out T);
  -- The value of X (resp. Y) after the call depends only
  -- on the value of Y (resp. X) before the call

  X : Natural;
  procedure Set_X_To_Zero;
  -- The value of X after the call depends on no input

end Show_Depends_Contracts;
```

Like Global contracts, you specify a Depends contract in subprogram declarations using an aspect. Its value is a list of one or more dependency relations between the outputs and inputs of the subprogram. Each relation is represented as two lists of variable names separated by an arrow. On the left of each arrow are variables whose final value depends on the initial value of the variables you list on the right.

For example, here we indicate that the final value of each parameter of `Swap` depends only on the initial value of the other parameter. If the subprogram is a function, we list its result as an output, using the `Result` attribute, as we do for `Get_Value_Of_X` below.

¹¹ https://docs.adacore.com/live/wave/spark2014/html/spark2014_ug/en/source/subprogram_contracts.html#data-dependencies

Listing 8: show_depends_contracts.ads

```

1 package Show_Depends_Contracts is
2
3   type T is new Integer;
4
5   X, Y, Z : T := 0;
6
7   procedure Swap (X, Y : in out T) with
8     Depends => (X => Y,
9                 -- X depends on the initial value of Y
10                Y => X);
11                 -- Y depends on the initial value of X
12
13   function Get_Value_Of_X return T with
14     Depends => (Get_Value_Of_X'Result => X);
15                 -- result depends on the initial value of X
16
17   procedure Set_X_To_Y_Plus_Z with
18     Depends => (X => (Y, Z));
19                 -- X depends on the initial values of Y and Z
20
21   procedure Set_X_To_X_Plus_Y with
22     Depends => (X =>+ Y);
23                 -- X depends on Y and X's initial value
24
25   procedure Do_Nothing (X : T) with
26     Depends => (null => X);
27                 -- no output is affected by X
28
29   procedure Set_X_To_Zero with
30     Depends => (X => null);
31                 -- X depends on no input
32
33 end Show_Depends_Contracts;

```

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Depends_Contracts
MD5: 290866c4208b6deff717a402bc2aef34

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...

Often, the final value of a variable depends on its own initial value. You can specify this in a concise way using the + character, as we did in the specification of `Set_X_To_X_Plus_Y` above. If there's more than one variable on the left of the arrow, a + means each variables depends on itself, not that they all depend on each other. You can write the corresponding dependency with `(=> +)` or without `(=>+)` whitespace.

If you have a program where an input isn't used to compute the final value of any output, you express that by writing `null` on the left of the dependency relation, as we did for the `Do_Nothing` subprogram above. You can only write one such dependency relation, which lists all unused inputs of the subprogram, and it must be written last. Such an annotation also silences flow analysis' warning about unused parameters. You can also write `null` on the right of a dependency relation to indicate that an output doesn't depend on any input. We do that above for the procedure `Set_X_To_Zero`.

Note: For more details on depends contracts, see the [SPARK User's Guide](#)¹².

2.4 Shortcomings

2.4.1 Modularity

Flow analysis is sound, meaning that if it doesn't output a message on some analyzed SPARK code, you can be assured that none of the errors it tests for can occur in that code. On the other hand, flow analysis often issues messages when there are, in fact, no errors. The first, and probably most common reason for this relates to modularity.

To scale flow analysis to large projects, verifications are usually done on a per-subprogram basis, including detection of uninitialized variables. To analyze this modularly, flow analysis needs to assume the initialization of inputs on subprogram entry and modification of outputs during subprogram execution. Therefore, each time a subprogram is called, flow analysis checks that global and parameter inputs are initialized and each time a subprogram returns, it checks that global and parameter outputs were modified.

This can produce error messages on perfectly correct subprograms. An example is `Set_X_To_Y_Plus_Z` below, which only sets its **out** parameter `X` when `Overflow` is **False**.

Listing 9: `set_x_to_y_plus_z.adb`

```
1 procedure Set_X_To_Y_Plus_Z
2   (Y, Z      : Natural;
3    X        : out Natural;
4    Overflow  : out Boolean)
5 is
6 begin
7   if Natural'Last - Z < Y then
8     Overflow := True; -- X should be initialized on every path
9   else
10    Overflow := False;
11    X := Y + Z;
12  end if;
13 end Set_X_To_Y_Plus_Z;
```

Code block metadata

Project: `Courses.Intro_To_Spark.Flow_Analysis.Set_X_To_Y_Plus_Z`
MD5: `be47cd769d2a7267c0bd1bb2ef0d6328`

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
set_x_to_y_plus_z.adb:3:04: medium: "X" might not be initialized in "Set_X_To_Y_
↳ Plus_Z" [reason for check: OUT parameter should be initialized on return]
↳ [possible fix: initialize "X" on all paths or make "X" an IN OUT parameter]
gnatprove: unproved check messages considered as errors
```

The message means that flow analysis wasn't able to verify that the program didn't read an uninitialized variable. To solve this problem, you can either set `X` to a dummy value when there's an overflow or manually verify that `X` is never used after a call to `Set_X_To_Y_Plus_Z` that returned **True** as the value of `Overflow`.

¹² https://docs.adacore.com/live/wave/spark2014/html/spark2014_ug/en/source/subprogram_contracts.html#flow-dependencies

2.4.2 Composite Types

Another common cause of false alarms is caused by the way flow analysis handles composite types. Let's start with arrays.

Flow analysis treats an entire array as single object instead of one object per element, so it considers modifying a single element to be a modification of the array as a whole. Obviously, this makes reasoning about which global variables are accessed less precise and hence the dependencies of those variables are also less precise. This also affects the ability to accurately detect reads of uninitialized data.

It's sometimes impossible for flow analysis to determine if an entire array object has been initialized. For example, after we write code to initialize every element of an unconstrained array *A* in chunks, we may still receive a message from flow analysis claiming that the array isn't initialized. To resolve this issue, you can either use a simpler loop over the full range of the array, or (even better) an aggregate assignment, or, if that's not possible, verify initialization of the object manually.

Listing 10: show_composite_types_shortcoming.ads

```

1 package Show_Composite_Types_Shortcoming is
2
3   type T is array (Natural range <>) of Integer;
4
5   procedure Init_Chunks (A : out T);
6   procedure Init_Loop (A : out T);
7   procedure Init_Aggregate (A : out T);
8
9 end Show_Composite_Types_Shortcoming;
```

Listing 11: show_composite_types_shortcoming.adb

```

1 package body Show_Composite_Types_Shortcoming is
2
3   procedure Init_Chunks (A : out T) is
4     begin
5       A (A'First) := 0;
6       for I in A'First + 1 .. A'Last loop
7         A (I) := 0;
8       end loop;
9       -- flow analysis doesn't know that A is initialized
10    end Init_Chunks;
11
12   procedure Init_Loop (A : out T) is
13     begin
14       for I in A'Range loop
15         A (I) := 0;
16       end loop;
17       -- flow analysis knows that A is initialized
18    end Init_Loop;
19
20   procedure Init_Aggregate (A : out T) is
21     begin
22       A := (others => 0);
23       -- flow analysis knows that A is initialized
24    end Init_Aggregate;
25
26 end Show_Composite_Types_Shortcoming;
```

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Composite_Types_Shortcoming
MD5: a366dcdd141191466027b2b928560c5e

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_composite_types_shortcoming.ads:5:27: medium: "A" might not be initialized in
↳ "Init_Chunks" [reason for check: OUT parameter should be fully initialized on
↳ return] [possible fix: initialize "A" on all paths, make "A" an IN OUT parameter
↳ or annotate it with aspect Relaxed_Initialization]
gnatprove: unproved check messages considered as errors
```

Flow analysis is more precise on record objects because it tracks the value of each component of a record separately within a single subprogram. So when a record object is initialized by successive assignments of its components, flow analysis knows that the entire object is initialized. However, record objects are still treated as single objects when analyzed as an input or output of a subprogram.

Listing 12: show_record_flow_analysis.ads

```
1 package Show_Record_Flow_Analysis is
2
3     type Rec is record
4         F1 : Natural;
5         F2 : Natural;
6     end record;
7
8     procedure Init (R : out Rec);
9
10 end Show_Record_Flow_Analysis;
```

Listing 13: show_record_flow_analysis.adb

```
1 package body Show_Record_Flow_Analysis is
2
3     procedure Init (R : out Rec) is
4     begin
5         R.F1 := 0;
6         R.F2 := 0;
7         -- R is initialized
8     end Init;
9
10 end Show_Record_Flow_Analysis;
```

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Record_Flow_Analysis_1
MD5: 24cd553b87b737536912b1bb780f6402

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_record_flow_analysis.ads:8:20: info: initialization of "R" proved
```

Flow analysis complains when a procedure call initializes only some components of a record object. It'll notify you of uninitialized components, as we see in subprogram Init_F2 below.

Listing 14: show_record_flow_analysis.ads

```

1 package Show_Record_Flow_Analysis is
2
3   type Rec is record
4     F1 : Natural;
5     F2 : Natural;
6   end record;
7
8   procedure Init (R : out Rec);
9   procedure Init_F2 (R : in out Rec);
10
11 end Show_Record_Flow_Analysis;
```

Listing 15: show_record_flow_analysis.adb

```

1 package body Show_Record_Flow_Analysis is
2
3   procedure Init_F2
4     (R : in out Rec) is
5   begin
6     R.F2 := 0;
7   end Init_F2;
8
9   procedure Init (R : out Rec) is
10  begin
11    R.F1 := 0;
12    Init_F2 (R); -- R should be initialized before this call
13  end Init;
14
15 end Show_Record_Flow_Analysis;
```

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Record_Flow_Analysis_2
MD5: efeecb787bf9d68977ed9701689cd6c4

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_record_flow_analysis.adb:12:16: high: "R.F2" is not initialized
gnatprove: unproved check messages considered as errors

2.4.3 Value Dependency

Flow analysis is not value-dependent: it never reasons about the values of expressions, only whether they have been set to some value or not. As a consequence, if some execution path in a subprogram is impossible, but the impossibility can only be determined by looking at the values of expressions, flow analysis still considers that path feasible and may emit messages based on it believing that execution along such a path is possible.

For example, in the version of `Absolute_Value` below, flow analysis computes that `R` is uninitialized on a path that enters neither of the two conditional statements. Because it doesn't consider values of expressions, it can't know that such a path is impossible.

Listing 16: absolute_value.adb

```
1 procedure Absolute_Value
2   (X : Integer;
3    R : out Natural)
4 is
5 begin
6   if X < 0 then
7     R := -X;
8   end if;
9   if X >= 0 then
10    R := X;
11  end if;
12  -- flow analysis doesn't know that R is initialized
13 end Absolute_Value;
```

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Absolute_Value_1
MD5: 69c233d22afdfdac679bf379b353a8d4

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
absolute_value.adb:3:04: medium: "R" might not be initialized in "Absolute_Value"
↳[reason for check: OUT parameter should be initialized on return] [possible fix:
↳initialize "R" on all paths or make "R" an IN OUT parameter]
gnatprove: unproved check messages considered as errors
```

To avoid this problem, you should make the control flow explicit, as in this second version of Absolute_Value:

Listing 17: absolute_value.adb

```
1 procedure Absolute_Value
2   (X : Integer;
3    R : out Natural)
4 is
5 begin
6   if X < 0 then
7     R := -X;
8   else
9     R := X;
10  end if;
11  -- flow analysis knows that R is initialized
12 end Absolute_Value;
```

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Absolute_Value_2
MD5: 9c773547f81e82a7aa1b45132b105937

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
```

2.4.4 Contract Computation

The final cause of unexpected flow messages that we'll discuss also comes from inaccuracy in computations of contracts. As we explained earlier, both Global and Depends contracts are optional, but GNATprove uses their data for some of its analysis.

For example, flow analysis can't detect reads from uninitialized variables without knowing the set of variables accessed. It needs to analyze and check both the Depends contracts you wrote for a subprogram and those you wrote for callers of that subprogram. Since each flow contract on a subprogram depends on the flow contracts of all the subprograms called inside its body, this computation can often be quite time-consuming. Therefore, flow analysis sometimes trades-off the precision of this computation against the time a more precise computation would take.

This is the case for Depends contracts, where flow analysis simply assumes the worst, that each subprogram's output depends on all of that subprogram's inputs. To avoid this assumption, all you have to do is supply contracts when default ones are not precise enough. You may also want to supply Global contracts to further speed up flow analysis on larger programs.

2.5 Code Examples / Pitfalls

2.5.1 Example #1

The procedure `Search_Array` searches for an occurrence of element `E` in an array `A`. If it finds one, it stores the index of the element in `Result`. Otherwise, it sets `Found` to **False**.

Listing 18: `show_search_array.ads`

```

1 package Show_Search_Array is
2
3   type Array_Of_Positives is array (Natural range <>) of Positive;
4
5   procedure Search_Array
6     (A      : Array_Of_Positives;
7      E      : Positive;
8      Result : out Integer;
9      Found  : out Boolean);
10
11 end Show_Search_Array;
```

Listing 19: `show_search_array.adb`

```

1 package body Show_Search_Array is
2
3   procedure Search_Array
4     (A      : Array_Of_Positives;
5      E      : Positive;
6      Result : out Integer;
7      Found  : out Boolean) is
8   begin
9     for I in A'Range loop
10      if A (I) = E then
11        Result := I;
12        Found  := True;
13        return;
14      end if;
15    end loop;
```

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```
16     Found := False;
17     end Search_Array;
18
19 end Show_Search_Array;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Flow_Analysis.Example_01
MD5: d2a27a5bde247767e2f6cd2d42a2d629
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_search_array.ads:8:07: medium: "Result" might not be initialized in "Search_
↳Array" [reason for check: OUT parameter should be initialized on return]
↳[possible fix: initialize "Result" on all paths or make "Result" an IN OUT
↳parameter]
gnatprove: unproved check messages considered as errors
```

GNATprove produces a message saying that Result is possibly uninitialized on return. There are perfectly legal uses of the function Search_Array, but flow analysis detects that Result is not initialized on the path that falls through from the loop. Even though this program is correct, you shouldn't ignore the message: it means flow analysis cannot guarantee that Result is always initialized at the call site and so assumes any read of Result at the call site will read initialized data. Therefore, you should either initialize Result when Found is false, which silences flow analysis, or verify this assumption at each call site by other means.

2.5.2 Example #2

To avoid the message previously issued by GNATprove, we modify Search_Array to raise an exception when E isn't found in A:

Listing 20: show_search_array.ads

```
1 package Show_Search_Array is
2
3     type Array_Of_Positives is array (Natural range <>) of Positive;
4
5     Not_Found : exception;
6
7     procedure Search_Array
8         (A      : Array_Of_Positives;
9          E      : Positive;
10         Result : out Integer);
11 end Show_Search_Array;
```

Listing 21: show_search_array.adb

```
1 package body Show_Search_Array is
2
3     procedure Search_Array
4         (A      : Array_Of_Positives;
5          E      : Positive;
6          Result : out Integer) is
7     begin
8         for I in A'Range loop
```

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```

9       if A (I) = E then
10          Result := I;
11          return;
12       end if;
13     end loop;
14     raise Not_Found;
15 end Search_Array;
16
17 end Show_Search_Array;

```

Code block metadata

```

Project: Courses.Intro_To_Spark.Flow_Analysis.Example_02
MD5: fa159faeb68974b1af3de2112e086b16

```

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_search_array.adb:14:07: medium: exception might be raised
gnatprove: unproved check messages considered as errors

```

Flow analysis doesn't emit any messages in this case, meaning it can verify that `Result` can't be read in SPARK code while uninitialized. But why is that, since `Result` is still not initialized when `E` is not in `A`? This is because the exception, `Not_Found`, can never be caught within SPARK code (SPAK doesn't allow exception handlers). However, the GNATprove tool also tries to ensure the absence of runtime errors in SPARK code, so tries to prove that `Not_Found` is never raised. When it can't do that here, it produces a different message.

2.5.3 Example #3

In this example, we're using a discriminated record for the result of `Search_Array` instead of conditionally raising an exception. By using such a structure, the place to store the index at which `E` was found exists only when `E` was indeed found. So if it wasn't found, there's nothing to be initialized.

Listing 22: show_search_array.ads

```

1 package Show_Search_Array is
2
3   type Array_Of_Positives is array (Natural range <>) of Positive;
4
5   type Search_Result (Found : Boolean := False) is record
6     case Found is
7       when True =>
8         Content : Integer;
9       when False => null;
10    end case;
11  end record;
12
13  procedure Search_Array
14    (A      : Array_Of_Positives;
15     E      : Positive;
16     Result : out Search_Result)
17  with Pre => not Result'Constrained;
18
19 end Show_Search_Array;

```

Listing 23: show_search_array.adb

```
1 package body Show_Search_Array is
2
3   procedure Search_Array
4     (A      : Array_Of_Positives;
5      E      : Positive;
6      Result : out Search_Result) is
7   begin
8     for I in A'Range loop
9       if A (I) = E then
10        Result := (Found => True,
11                  Content => I);
12        return;
13      end if;
14    end loop;
15    Result := (Found => False);
16  end Search_Array;
17
18 end Show_Search_Array;
```

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Example_03
MD5: 1d5ec5d78185fd75499b90b3d21f8ae2

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_search_array.adb:10:20: info: discriminant check proved
show_search_array.adb:15:14: info: discriminant check proved
show_search_array.ads:16:07: info: initialization of "Result" proved
```

This example is correct and flow analysis doesn't issue any message: it can verify both that no uninitialized variables are read in `Search_Array`'s body, and that all its outputs are set on return. We've used the attribute `Constrained` in the precondition of `Search_Array` to indicate that the value of the `Result` in argument can be set to any variant of the record type `Search_Result`, specifically to either the variant where `E` was found and where it wasn't.

2.5.4 Example #4

The function `Size_Of_Biggest_Increasing_Sequence` is supposed to find all sequences within its parameter `A` that contain elements with increasing values and returns the length of the longest one. To do this, it calls a nested procedure `Test_Index` iteratively on all the elements of `A`. `Test_Index` checks if the sequence is still increasing. If so, it updates the largest value seen so far in this sequence. If not, it means it's found the end of a sequence, so it computes the size of that sequence and stores it in `Size_Of_Seq`.

Listing 24: show_biggest_increasing_sequence.ads

```
1 package Show_Biggest_Increasing_Sequence is
2
3   type Array_Of_Positives is array (Integer range <>) of Positive;
4
5   function Size_Of_Biggest_Increasing_Sequence (A : Array_Of_Positives)
6     return Natural;
```

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```

7
8 end Show_Biggest_Increasing_Sequence;

```

Listing 25: show_biggest_increasing_sequence.adb

```

1 package body Show_Biggest_Increasing_Sequence is
2
3   function Size_Of_Biggest_Increasing_Sequence (A : Array_Of_Positives)
4     return Natural
5   is
6     Max      : Natural;
7     End_Of_Seq : Boolean;
8     Size_Of_Seq : Natural;
9     Beginning : Integer;
10
11    procedure Test_Index (Current_Index : Integer) is
12    begin
13      if A (Current_Index) >= Max then
14        Max := A (Current_Index);
15        End_Of_Seq := False;
16      else
17        Max      := 0;
18        End_Of_Seq := True;
19        Size_Of_Seq := Current_Index - Beginning;
20        Beginning := Current_Index;
21      end if;
22    end Test_Index;
23
24    Biggest_Seq : Natural := 0;
25
26    begin
27      for I in A'Range loop
28        Test_Index (I);
29        if End_Of_Seq then
30          Biggest_Seq := Natural'Max (Size_Of_Seq, Biggest_Seq);
31        end if;
32      end loop;
33      return Biggest_Seq;
34    end Size_Of_Biggest_Increasing_Sequence;
35
36 end Show_Biggest_Increasing_Sequence;

```

Code block metadata

```

Project: Courses.Intro_To_Spark.Flow_Analysis.Example_04
MD5: e6083665827d9dee4e00bdce4c1e962f

```

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_biggest_increasing_sequence.adb:13:34: medium: "Max" might not be initialized,
↳ in call inlined at show_biggest_increasing_sequence.adb:28
show_biggest_increasing_sequence.adb:19:44: medium: "Beginning" might not be
↳ initialized, in call inlined at show_biggest_increasing_sequence.adb:28
show_biggest_increasing_sequence.adb:30:41: medium: "Size_Of_Seq" might not be
↳ initialized
gnatprove: unproved check messages considered as errors

```

However, this example is not correct. Flow analysis emits messages for `Test_Index` stating that `Max`, `Beginning`, and `Size_Of_Seq` should be initialized before being read. Indeed,

when you look carefully, you see that both `Max` and `Beginning` are missing initializations because they are read in `Test_Index` before being written. As for `Size_Of_Seq`, we only read its value when `End_Of_Seq` is true, so it actually can't be read before being written, but flow analysis isn't able to verify its initialization by using just flow information.

The call to `Test_Index` is automatically inlined by GNATprove, which leads to another messages above. If GNATprove couldn't inline the call to `Test_Index`, for example if it was defined in another unit, the same messages would be issued on the call to `Test_Index`.

2.5.5 Example #5

In the following example, we model permutations as arrays where the element at index `I` is the position of the `I`'th element in the permutation. The procedure `Init` initializes a permutation to the identity, where the `I`'th elements is at the `I`'th position. `Cyclic_Permutation` calls `Init` and then swaps elements to construct a cyclic permutation.

Listing 26: show_permutation.ads

```
1 package Show_Permutation is
2
3   type Permutation is array (Positive range <>) of Positive;
4
5   procedure Swap (A : in out Permutation;
6                 I, J : Positive);
7
8   procedure Init (A : out Permutation);
9
10  function Cyclic_Permutation (N : Natural) return Permutation;
11
12 end Show_Permutation;
```

Listing 27: show_permutation.adb

```
1 package body Show_Permutation is
2
3   procedure Swap (A : in out Permutation;
4                 I, J : Positive)
5   is
6     Tmp : Positive := A (I);
7   begin
8     A (I) := A (J);
9     A (J) := Tmp;
10  end Swap;
11
12  procedure Init (A : out Permutation) is
13  begin
14    A (A'First) := A'First;
15    for I in A'First + 1 .. A'Last loop
16      A (I) := I;
17    end loop;
18  end Init;
19
20  function Cyclic_Permutation (N : Natural) return Permutation is
21  A : Permutation (1 .. N);
22  begin
23    Init (A);
24    for I in A'First .. A'Last - 1 loop
25      Swap (A, I, I + 1);
26    end loop;
27    return A;
```

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```

28   end Cyclic_Permutation;
29
30 end Show_Permutation;

```

Code block metadata

```

Project: Courses.Intro_To_Spark.Flow_Analysis.Example_05
MD5: 219b06617c636c18543128d77f90fcee

```

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_permutation.ads:8:20: medium: "A" might not be initialized in "Init" [reason:
↳for check: OUT parameter should be fully initialized on return] [possible fix:
↳initialize "A" on all paths, make "A" an IN OUT parameter or annotate it with
↳aspect Relaxed_Initialization]
gnatprove: unproved check messages considered as errors

```

This program is correct. However, flow analysis will nevertheless still emit messages because it can't verify that every element of A is initialized by the loop in Init. This message is a false alarm. You can either ignore it or justify it safely.

2.5.6 Example #6

This program is the same as the previous one except that we've changed the mode of A in the specification of Init to **in out** to avoid the message from flow analysis on array assignment.

Listing 28: show_permutation.ads

```

1 package Show_Permutation is
2
3   type Permutation is array (Positive range <>) of Positive;
4
5   procedure Swap (A : in out Permutation;
6                 I, J : Positive);
7
8   procedure Init (A : in out Permutation);
9
10  function Cyclic_Permutation (N : Natural) return Permutation;
11
12 end Show_Permutation;

```

Listing 29: show_permutation.adb

```

1 package body Show_Permutation is
2
3   procedure Swap (A : in out Permutation;
4                 I, J : Positive)
5   is
6     Tmp : Positive := A (I);
7   begin
8     A (I) := A (J);
9     A (J) := Tmp;
10  end Swap;
11
12  procedure Init (A : in out Permutation) is

```

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```

13  begin
14      A (A'First) := A'First;
15      for I in A'First + 1 .. A'Last loop
16          A (I) := I;
17      end loop;
18  end Init;
19
20  function Cyclic_Permutation (N : Natural) return Permutation is
21      A : Permutation (1 .. N);
22  begin
23      Init (A);
24      for I in A'First .. A'Last - 1 loop
25          Swap (A, I, I + 1);
26      end loop;
27      return A;
28  end Cyclic_Permutation;
29
30  end Show_Permutation;

```

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Example_06
MD5: 61406d9a66dda71630c74c12f3d67936

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_permutation.adb:23:13: high: "A" is not initialized
gnatprove: unproved check messages considered as errors

This program is not correct. Changing the mode of a parameter that should really be **out** to **in out** to silence a false alarm is not a good idea. Not only does this obfuscate the specification of `Init`, but flow analysis emits a message on the procedure where `A` is not initialized, as shown by the message in `Cyclic_Permutation`.

2.5.7 Example #7

`Incr_Step_Function` takes an array `A` as an argument and iterates through `A` to increment every element by the value of `Increment`, saturating at a specified threshold value. We specified a Global contract for `Incr_Until_Threshold`.

Listing 30: show_increments.ads

```

1  package Show_Increments is
2
3      type Array_Of_Positives is array (Natural range <>) of Positive;
4
5      Increment : constant Natural := 10;
6
7      procedure Incr_Step_Function (A : in out Array_Of_Positives);
8
9  end Show_Increments;

```

Listing 31: show_increments.adb

```

1  package body Show_Increments is
2

```

(continues on next page)

(continued from previous page)

```

3  procedure Incr_Step_Function (A : in out Array_Of_Positives) is
4
5      Threshold : Positive := Positive'Last;
6
7      procedure Incr_Until_Threshold (I : Integer) with
8          Global => (Input => Threshold,
9                  In_Out => A);
10
11     procedure Incr_Until_Threshold (I : Integer) is
12     begin
13         if Threshold - Increment <= A (I) then
14             A (I) := Threshold;
15         else
16             A (I) := A (I) + Increment;
17         end if;
18     end Incr_Until_Threshold;
19
20     begin
21         for I in A'Range loop
22             if I > A'First then
23                 Threshold := A (I - 1);
24             end if;
25             Incr_Until_Threshold (I);
26         end loop;
27     end Incr_Step_Function;
28
29 end Show_Increments;

```

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Example_07
MD5: 8e28a005cd9d78947e4bfc60db708bf5

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_increments.adb:8:09: info: data dependencies proved

Everything is fine here. Specifically, the Global contract is correct. It mentions both Threshold, which is read but not written in the procedure, and A, which is both read and written. The fact that A is a parameter of an enclosing unit doesn't prevent us from using it inside the Global contract; it really is global to Incr_Until_Threshold. We didn't mention Increment since it's a static constant.

2.5.8 Example #8

We now go back to the procedure Test_Index from [Example #4](#) (page 36) and correct the missing initializations. We want to know if the Global contract of Test_Index is correct.

Listing 32: show_biggest_increasing_sequence.ads

```

1  package Show_Biggest_Increasing_Sequence is
2
3      type Array_Of_Positives is array (Integer range <>) of Positive;
4
5      function Size_Of_Biggest_Increasing_Sequence (A : Array_Of_Positives)
6          return Natural;

```

(continues on next page)

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```

7
8 end Show_Biggest_Increasing_Sequence;

```

Listing 33: show_biggest_increasing_sequence.adb

```

1 package body Show_Biggest_Increasing_Sequence is
2
3   function Size_Of_Biggest_Increasing_Sequence (A : Array_Of_Positives)
4     return Natural
5   is
6     Max      : Natural := 0;
7     End_Of_Seq : Boolean;
8     Size_Of_Seq : Natural := 0;
9     Beginning : Integer := A'First - 1;
10
11    procedure Test_Index (Current_Index : Integer) with
12      Global => (In_Out => (Beginning, Max, Size_Of_Seq),
13        Output => End_Of_Seq,
14        Input  => Current_Index)
15    is
16      begin
17        if A (Current_Index) >= Max then
18          Max := A (Current_Index);
19          End_Of_Seq := False;
20        else
21          Max      := 0;
22          End_Of_Seq := True;
23          Size_Of_Seq := Current_Index - Beginning;
24          Beginning := Current_Index;
25        end if;
26      end Test_Index;
27
28      Biggest_Seq : Natural := 0;
29
30    begin
31      for I in A'Range loop
32        Test_Index (I);
33        if End_Of_Seq then
34          Biggest_Seq := Natural'Max (Size_Of_Seq, Biggest_Seq);
35        end if;
36      end loop;
37      return Biggest_Seq;
38    end Size_Of_Biggest_Increasing_Sequence;
39
40 end Show_Biggest_Increasing_Sequence;

```

Code block metadata

```

Project: Courses.Intro_To_Spark.Flow_Analysis.Example_08
MD5: 86fb934c32a38f6841ef736780b2e3b2

```

Prover output

```

Phase 1 of 2: generation of Global contracts ...
show_biggest_increasing_sequence.adb:14:30: error: global item cannot reference_
↳parameter of subprogram "Test_Index"
gnatprove: error during generation of Global contracts

```

The contract in this example is not correct: `Current_Index` is a parameter of `Test_Index`, so we shouldn't reference it as a global variable. Also, we should have listed variable `A` from the outer scope as an `Input` in the `Global` contract.

2.5.9 Example #9

Next, we change the Global contract of Test_Index into a Depends contract. In general, we don't need both contracts because the set of global variables accessed can be deduced from the Depends contract.

Listing 34: show_biggest_increasing_sequence.ads

```

1 package Show_Biggest_Increasing_Sequence is
2
3   type Array_Of_Positives is array (Integer range <>) of Positive;
4
5   function Size_Of_Biggest_Increasing_Sequence (A : Array_Of_Positives)
6     return Natural;
7
8 end Show_Biggest_Increasing_Sequence;
```

Listing 35: show_biggest_increasing_sequence.adb

```

1 package body Show_Biggest_Increasing_Sequence is
2
3   function Size_Of_Biggest_Increasing_Sequence (A : Array_Of_Positives)
4     return Natural
5   is
6     Max      : Natural := 0;
7     End_Of_Seq : Boolean;
8     Size_Of_Seq : Natural := 0;
9     Beginning  : Integer := A'First - 1;
10
11    procedure Test_Index (Current_Index : Integer) with
12      Depends => ((Max, End_Of_Seq) => (A, Current_Index, Max),
13                (Size_Of_Seq, Beginning) =>
14                  + (A, Current_Index, Max, Beginning))
15    is
16    begin
17      if A (Current_Index) >= Max then
18        Max := A (Current_Index);
19        End_Of_Seq := False;
20      else
21        Max      := 0;
22        End_Of_Seq := True;
23        Size_Of_Seq := Current_Index - Beginning;
24        Beginning  := Current_Index;
25      end if;
26    end Test_Index;
27
28    Biggest_Seq : Natural := 0;
29
30    begin
31      for I in A'Range loop
32        Test_Index (I);
33        if End_Of_Seq then
34          Biggest_Seq := Natural'Max (Size_Of_Seq, Biggest_Seq);
35        end if;
36      end loop;
37      return Biggest_Seq;
38    end Size_Of_Biggest_Increasing_Sequence;
39
40 end Show_Biggest_Increasing_Sequence;
```

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Example_09
MD5: d54ac5d4266738b1bf64869131644b33

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_biggest_increasing_sequence.adb:7:07: info: initialization of "End_Of_Seq"
↳proved
show_biggest_increasing_sequence.adb:11:17: info: initialization of "End_Of_Seq"
↳proved
show_biggest_increasing_sequence.adb:12:09: info: flow dependencies proved
```

This example is correct. Some of the dependencies, such as `Size_Of_Seq` depending on `Beginning`, come directly from the assignments in the subprogram. Since the control flow influences the final value of all of the outputs, the variables that are being read, `A`, `Current_Index`, and `Max`, are present in every dependency relation. Finally, the dependencies of `Size_Of_Seq` and `Beginning` on themselves are because they may not be modified by the subprogram execution.

2.5.10 Example #10

The subprogram `Identity` swaps the value of its parameter two times. Its `Depends` contract says that the final value of `X` only depends on its initial value and likewise for `Y`.

Listing 36: show_swap.ads

```
1 package Show_Swap is
2
3   procedure Swap (X, Y : in out Positive);
4
5   procedure Identity (X, Y : in out Positive) with
6     Depends => (X => X,
7                Y => Y);
8
9 end Show_Swap;
```

Listing 37: show_swap.adb

```
1 package body Show_Swap is
2
3   procedure Swap (X, Y : in out Positive) is
4     Tmp : constant Positive := X;
5   begin
6     X := Y;
7     Y := Tmp;
8   end Swap;
9
10  procedure Identity (X, Y : in out Positive) is
11  begin
12    Swap (X, Y);
13    Swap (Y, X);
14  end Identity;
15
16 end Show_Swap;
```

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Example_10
MD5: 8567ece1e5bbc190f62dd483785d078a

Prover output

```
Phase 1 of 2: generation of Global contracts ...  
Phase 2 of 2: analysis of data and information flow ...  
show_swap.ads:6:18: medium: missing dependency "X => Y"  
show_swap.ads:7:18: medium: missing dependency "Y => X"  
gnatprove: unproved check messages considered as errors
```

This code is correct, but flow analysis can't verify the Depends contract of Identity because we didn't supply a Depends contract for Swap. Therefore, flow analysis assumes that all outputs of Swap, X and Y, depend on all its inputs, both X and Y's initial values. To prevent this, we should manually specify a Depends contract for Swap.

PROOF OF PROGRAM INTEGRITY

This section presents the proof capability of GNATprove, a major tool for the SPARK language. We focus here on the simpler proofs that you'll need to write to verify your program's integrity. The primary objective of performing proof of your program's integrity is to ensure the absence of runtime errors during its execution.

The analysis steps discussed here are only sound if you've previously performed *Flow Analysis* (page 21). You shouldn't proceed further if you still have unjustified flow analysis messages for your program.

3.1 Runtime Errors

There's always the potential for errors that aren't detected during compilation to occur during a program's execution. These errors, called runtime errors, are those targeted by GNATprove.

There are various kinds of runtime errors, the most common being references that are out of the range of an array (*buffer overflow*¹³ in Ada), subtype range violations, overflows in computations, and divisions by zero. The code below illustrates many examples of possible runtime errors, all within a single statement. Look at the assignment statement setting the $I + J$ 'th cell of an array A to the value P / Q .

Listing 1: show_runtime_errors.ads

```
1 package Show_Runtime_Errors is
2
3     type Nat_Array is array (Integer range <>) of Natural;
4
5     procedure Update (A : in out Nat_Array; I, J, P, Q : Integer);
6
7 end Show_Runtime_Errors;
```

Listing 2: show_runtime_errors.adb

```
1 package body Show_Runtime_Errors is
2
3     procedure Update (A : in out Nat_Array; I, J, P, Q : Integer) is
4     begin
5         A (I + J) := P / Q;
6     end Update;
7
8 end Show_Runtime_Errors;
```

Code block metadata

¹³ https://en.wikipedia.org/wiki/Buffer_overflow

Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Runtime_Errors
MD5: c0718b8cb6138b84a99e0040e2a9164e

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_runtime_errors.adb:5:12: medium: overflow check might fail, cannot prove
↳ lower bound for I + J [reason for check: result of addition must fit in a 32-
↳ bits machine integer] [possible fix: add precondition (if J >= 0 then I <=
↳ Integer'Last - J else I >= Integer'First - J) to subprogram at show_runtime_
↳ errors.ads:5]
show_runtime_errors.adb:5:12: medium: array index check might fail [reason for
↳ check: result of addition must be a valid index into the array] [possible fix:
↳ add precondition (if J >= 0 then I <= A'Last - J else I >= A'First - J) to
↳ subprogram at show_runtime_errors.ads:5]
show_runtime_errors.adb:5:22: medium: divide by zero might fail [possible fix: add
↳ precondition (Q /= 0) to subprogram at show_runtime_errors.ads:5]
show_runtime_errors.adb:5:22: medium: overflow check might fail, cannot prove
↳ lower bound for P / Q [reason for check: result of division must fit in a 32-
↳ bits machine integer] [possible fix: add precondition (P / Q in Integer) to
↳ subprogram at show_runtime_errors.ads:5]
show_runtime_errors.adb:5:22: medium: range check might fail, cannot prove lower
↳ bound for P / Q [reason for check: result of division must fit in the target
↳ type of the assignment] [possible fix: add precondition (P / Q in Natural) to
↳ subprogram at show_runtime_errors.ads:5]
gnatprove: unproved check messages considered as errors
```

There are quite a number of errors that may occur when executing this code. If we don't know anything about the values of I, J, P, and Q, we can't rule out any of those errors.

First, the computation of $I + J$ can overflow, for example if I is `Integer'Last` and J is positive.

```
A (Integer'Last + 1) := P / Q;
```

Next, the sum, which is used as an array index, may not be in the range of the index of the array.

```
A (A'Last + 1) := P / Q;
```

On the other side of the assignment, the division may also overflow, though only in the very special case where P is `Integer'First` and Q is -1 because of the asymmetric range of signed integer types.

```
A (I + J) := Integer'First / -1;
```

The division is also not allowed if Q is 0.

```
A (I + J) := P / 0;
```

Finally, since the array contains natural numbers, it's also an error to store a negative value in it.

```
A (I + J) := 1 / -1;
```

The compiler generates checks in the executable code corresponding to each of those runtime errors. Each check raises an exception if it fails. For the above assignment statement, we can see examples of exceptions raised due to failed checks for each of the different cases above.

```

A (Integer'Last + 1) := P / Q;
-- raised CONSTRAINT_ERROR : overflow check failed

A (A'Last + 1) := P / Q;
-- raised CONSTRAINT_ERROR : index check failed

A (I + J) := Integer'First / (-1);
-- raised CONSTRAINT_ERROR : overflow check failed

A (I + J) := 1 / (-1);
-- raised CONSTRAINT_ERROR : range check failed

A (I + J) := P / 0;
-- raised CONSTRAINT_ERROR : divide by zero

```

These runtime checks are costly, both in terms of program size and execution time. It may be appropriate to remove them if we can statically ensure they aren't needed at runtime, in other words if we can prove that the condition tested for can never occur.

This is where the analysis done by GNATprove comes in. It can be used to demonstrate statically that none of these errors can ever occur at runtime. Specifically, GNATprove logically interprets the meaning of every instruction in the program. Using this interpretation, GNATprove generates a logical formula called a *verification condition* for each check that would otherwise be required by the Ada (and hence SPARK) language.

```

A (Integer'Last + 1) := P / Q;
-- medium: overflow check might fail

A (A'Last + 1) := P / Q;
-- medium: array index check might fail

A (I + J) := Integer'First / (-1);
-- medium: overflow check might fail

A (I + J) := 1 / (-1);
-- medium: range check might fail

A (I + J) := P / 0;
-- medium: divide by zero might fail

```

GNATprove then passes these verification conditions to an automatic prover, stated as conditions that must be true to avoid the error. If every such condition can be validated by a prover (meaning that it can be mathematically shown to always be true), we've been able to prove that no error can ever be raised at runtime when executing that program.

3.2 Modularity

To scale to large programs, GNATprove performs proofs on a per-subprogram basis by relying on preconditions and postconditions to properly summarize the input and output state of each subprogram. More precisely, when verifying the body of a subprogram, GNATprove assumes it knows nothing about the possible initial values of its parameters and of the global variables it accesses except what you state in the subprogram's precondition. If you don't specify a precondition, it can't make any assumptions.

For example, the following code shows that the body of `Increment` can be successfully verified: its precondition constrains the value of its parameter `X` to be less than `Integer'Last` so we know the overflow check is always false.

In the same way, when a subprogram is called, GNATprove assumes its **out** and **in out** parameters and the global variables it writes can be modified in any way compatible with their postconditions. For example, since `Increment` has no postcondition, GNATprove doesn't know that the value of `X` after the call is always less than `Integer'Last`. Therefore, it can't prove that the addition following the call to `Increment` can't overflow.

Listing 3: show_modularity.adb

```
1 procedure Show_Modularity is
2
3   procedure Increment (X : in out Integer) with
4     Pre => X < Integer'Last is
5   begin
6     X := X + 1;
7     -- info: overflow check proved
8   end Increment;
9
10  X : Integer;
11 begin
12  X := Integer'Last - 2;
13  Increment (X);
14  -- After the call, GNATprove no longer knows the value of X
15
16  X := X + 1;
17  -- medium: overflow check might fail
18 end Show_Modularity;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Modularity_1
MD5: ca8ff8d29792fd5a06f7cb0158e13689
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_modularity.adb:6:14: info: overflow check proved
show_modularity.adb:10:04: info: initialization of "X" proved
show_modularity.adb:13:04: info: precondition proved
show_modularity.adb:16:11: medium: overflow check might fail, cannot prove upper
↳bound for X + 1 [reason for check: result of addition must fit in a 32-bits
↳machine integer] [possible fix: call at line 13 should mention X (for argument
↳X) in a postcondition]
gnatprove: unproved check messages considered as errors
```

3.2.1 Exceptions

There are two cases where GNATprove doesn't require modularity and hence doesn't make the above assumptions. First, local subprograms without contracts can be inlined if they're simple enough and are neither recursive nor have multiple return points. If we remove the contract from `Increment`, it fits the criteria for inlining.

Listing 4: show_modularity.adb

```
1 procedure Show_Modularity is
2
3   procedure Increment (X : in out Integer) is
4   begin
5     X := X + 1;
6     -- info: overflow check proved, in call inlined at...
```

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```

7   end Increment;
8
9   X : Integer;
10  begin
11   X := Integer'Last - 2;
12   Increment (X);
13   X := X + 1;
14   -- info: overflow check proved
15  end Show_Modularity;

```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Modularity_2
MD5: 448d576897c3e4606cd4b90621aad63a

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_modularity.adb:5:14: info: overflow check proved, in call inlined at show_
↳modularity.adb:12
show_modularity.adb:9:04: info: initialization of "X" proved
show_modularity.adb:13:11: info: overflow check proved

```

GNATprove now sees the call to Increment exactly as if the increment on X was done outside that call, so it can successfully verify that neither addition can overflow.

Note: For more details on contextual analysis of subprograms, see the [SPARK User's Guide](#)¹⁴.

The other case involves functions. If we define a function as an expression function, with or without contracts, GNATprove uses the expression itself as the postcondition on the result of the function.

In our example, replacing Increment with an expression function allows GNATprove to successfully verify the overflow check in the addition.

Listing 5: show_modularity.adb

```

1  procedure Show_Modularity is
2
3     function Increment (X : Integer) return Integer is
4         (X + 1)
5         -- info: overflow check proved
6         with Pre => X < Integer'Last;
7
8     X : Integer;
9  begin
10   X := Integer'Last - 2;
11   X := Increment (X);
12   X := X + 1;
13   -- info: overflow check proved
14  end Show_Modularity;

```

Code block metadata

¹⁴ https://docs.adacore.com/live/wave/spark2014/html/spark2014_ug/en/source/how_to_write_subprogram_contracts.html#contextual-analysis-of-subprograms-without-contracts

```
Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Modularity_3
MD5: b2b67845362929472e4e23867fcbd5e7
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_modularity.adb:4:09: info: overflow check proved
show_modularity.adb:8:04: info: initialization of "X" proved
show_modularity.adb:11:09: info: precondition proved
show_modularity.adb:12:11: info: overflow check proved
```

Note: For more details on expression functions, see the [SPARK User's Guide¹⁵](#).

3.3 Contracts

Ada contracts are perfectly suited for formal verification, but are primarily designed to be checked at runtime. When you specify the `-gnata` switch, the compiler generates code that verifies the contracts at runtime. If an Ada contract isn't satisfied for a given subprogram call, the program raises the `Assert_Failure` exception. This switch is particularly useful during development and testing, but you may also retain run-time execution of assertions, and specifically preconditions, during the program's deployment to avoid an inconsistent state.

Consider the incorrect call to `Increment` below, which violates its precondition. One way to detect this error is by compiling the function with assertions enabled and testing it with inputs that trigger the violation. Another way, one that doesn't require guessing the needed inputs, is to run `GNATprove`.

Listing 6: `show_precondition_violation.adb`

```
1 procedure Show_Precondition_Violation is
2
3   procedure Increment (X : in out Integer) with
4     Pre => X < Integer'Last is
5     begin
6       X := X + 1;
7     end Increment;
8
9   X : Integer;
10
11 begin
12   X := Integer'Last;
13   Increment (X);
14 end Show_Precondition_Violation;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Precondition_Violation
MD5: 60cb889128fc6bca10e21b1baf041258
```

Prover output

¹⁵ https://docs.adacore.com/live/wave/spark2014/html/spark2014_ug/en/source/specification_features.html#expression-functions

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_precondition_violation.adb:13:04: medium: precondition might fail
gnatprove: unproved check messages considered as errors
```

Runtime output

```
raised ADA.ASSERTIONS.ASSERTION_ERROR : failed precondition from show_precondition_
↳violation.adb:4
```

Similarly, consider the incorrect implementation of function `Absolute` below, which violates its postcondition. Likewise, one way to detect this error is by compiling the function with assertions enabled and testing with inputs that trigger the violation. Another way, one which again doesn't require finding the inputs needed to demonstrate the error, is to run GNATprove.

Listing 7: `show_postcondition_violation.adb`

```
1 procedure Show_Postcondition_Violation is
2
3   procedure Absolute (X : in out Integer) with
4     Post => X >= 0 is
5   begin
6     if X > 0 then
7       X := -X;
8     end if;
9   end Absolute;
10
11   X : Integer;
12
13 begin
14   X := 1;
15   Absolute (X);
16 end Show_Postcondition_Violation;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Postcondition_Violation
MD5: fb1340de7e082d801f177bd8a0cf90a6
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_postcondition_violation.adb:4:14: medium: postcondition might fail
gnatprove: unproved check messages considered as errors
```

Runtime output

```
raised ADA.ASSERTIONS.ASSERTION_ERROR : failed postcondition from show_
↳postcondition_violation.adb:4
```

The benefits of dynamically checking contracts extends beyond making testing easier. Early failure detection also allows an easier recovery and facilitates debugging, so you may want to enable these checks at runtime to terminate execution before some damaging or hard-to-debug action occurs.

GNATprove statically analyses preconditions and postconditions. It verifies preconditions every time a subprogram is called, which is the runtime semantics of contracts. Postconditions, on the other hand, are verified once as part of the verification of the subprogram's

body. For example, GNATprove must wait until `Increment` is improperly called to detect the precondition violation, since a precondition is really a contract for the caller. On the other hand, it doesn't need `Absolute` to be called to detect that its postcondition doesn't hold for all its possible inputs.

Note: For more details on pre and postconditions, see the [SPARK User's Guide](#)¹⁶.

3.3.1 Executable Semantics

Expressions in Ada contracts have the same semantics as Boolean expressions elsewhere, so runtime errors can occur during their computation. To simplify both debugging of assertions and combining testing and static verification, the same semantics are used by GNATprove.

While proving programs, GNATprove verifies that no error can ever be raised during the execution of the contracts. However, you may sometimes find those semantics too heavy, in particular with respect to overflow checks, because they can make it harder to specify an appropriate precondition. We see this in the function `Add` below.

Listing 8: `show_executable_semantics.adb`

```
1 procedure Show_Executable_Semantics
2   with SPARK_Mode => On
3 is
4   function Add (X, Y : Integer) return Integer is (X + Y)
5     with Pre => X + Y in Integer;
6
7   X : Integer;
8 begin
9   X := Add (Integer'Last, 1);
10 end Show_Executable_Semantics;
```

Code block metadata

Project: `Courses.Intro_To_Spark.Proof_of_Program_Integrity.Executable_Semantics`
MD5: `d85fa0507d7c35fb98ade7815020117e`

Build output

```
show_executable_semantics.adb:5:24: warning: explicit membership test may be
↳ optimized away [enabled by default]
show_executable_semantics.adb:5:24: warning: use 'Valid attribute instead [enabled
↳ by default]
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_executable_semantics.adb:5:20: medium: overflow check might fail, cannot
↳ prove lower bound for X + Y [reason for check: result of addition must fit in a
↳ 32-bits machine integer] [possible fix: use pragma Overflow_Mode or switch -
↳ gnato13 or unit Ada.Numerics.Big_Numerics.Big_Integers]
show_executable_semantics.adb:9:09: medium: precondition might fail, cannot prove
↳ upper bound for Add (Integer'Last, 1)
gnatprove: unproved check messages considered as errors
```

Runtime output

¹⁶ https://docs.adacore.com/live/wave/spark2014/html/spark2014_ug/en/source/subprogram_contracts.html#preconditions

```
raised CONSTRAINT_ERROR : show_executable_semantics.adb:5 overflow check failed
```

GNATprove issues a message on this code warning about a possible overflow when computing the sum of X and Y in the precondition. Indeed, since expressions in assertions have normal Ada semantics, this addition can overflow, as you can easily see by compiling and running the code that calls `Add` with arguments `Integer'Last` and `1`.

On the other hand, you sometimes may prefer GNATprove to use the mathematical semantics of addition in contracts while the generated code still properly verifies that no error is ever raised at runtime in the body of the program. You can get this behavior by using the compiler switch `-gnato??` (for example `-gnato13`), which allows you to independently set the overflow mode in code (the first digit) and assertions (the second digit). For both, you can either reduce the number of overflow checks (the value `2`), completely eliminate them (the value `3`), or preserve the default Ada semantics (the value `1`).

Note: For more details on overflow modes, see the [SPARK User's Guide](#)¹⁷.

3.3.2 Additional Assertions and Contracts

As we've seen, a key feature of SPARK is that it allows us to state properties to check using assertions and contracts. SPARK supports preconditions and postconditions as well as assertions introduced by the `Assert` pragma.

The SPARK language also includes new contract types used to assist formal verification. The new pragma `Assume` is treated as an assertion during execution but introduces an assumption when proving programs. Its value is a Boolean expression which GNATprove assumes to be true without any attempt to verify that it's true. You'll find this feature useful, but you must use it with great care. Here's an example of using it.

Listing 9: `incr.adb`

```
1 procedure Incr (X : in out Integer) is
2 begin
3   pragma Assume (X < Integer'Last);
4   X := X + 1;
5 end Incr;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Pragma_Assume
MD5: bfb4b8aca259d7516b6acaee571f8c2
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
incr.adb:4:11: info: overflow check proved
```

Note: For more details on pragma `Assume`, see the [SPARK User's Guide](#)¹⁸.

The `Contract_Cases` aspect is another construct introduced for GNATprove, but which also acts as an assertion during execution. It allows you to specify the behavior of a subprogram

¹⁷ https://docs.adacore.com/live/wave/spark2014/html/spark2014_ug/en/source/overflow_modes.html

¹⁸ https://docs.adacore.com/live/wave/spark2014/html/spark2014_ug/en/source/assertion_pragmas.html#pragma-assume

using a disjunction of cases. Each element of a `Contract_Cases` aspect is a *guard*, which is evaluated before the call and may only reference the subprogram's inputs, and a *consequence*. At each call of the subprogram, one and only one guard is permitted to evaluate to **True**. The consequence of that case is a contract that's required to be satisfied when the subprogram returns.

Listing 10: absolute.adb

```
1 procedure Absolute (X : in out Integer) with
2   Pre           => X > Integer'First,
3   Contract_Cases => (X < 0 => X = -X'0ld,
4                     X >= 0 => X = X'0ld)
5 is
6 begin
7   if X < 0 then
8     X := -X;
9   end if;
10 end Absolute;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Absolute
MD5: 5ac868f35be18bb6fffe2444ecbea28d
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
absolute.adb:3:03: info: disjoint contract cases proved
absolute.adb:3:03: info: complete contract cases proved
absolute.adb:3:29: info: contract case proved
absolute.adb:3:36: info: overflow check proved
absolute.adb:4:29: info: contract case proved
absolute.adb:8:12: info: overflow check proved
```

Similarly to how it analyzes a subprogram's precondition, GNATprove verifies the `Contract_Cases` only once. It verifies the validity of each consequence (given the truth of its guard) and the disjointness and completeness of the guard conditions (meaning that exactly one guard must be true for each possible set of input values).

Note: For more details on `Contract_Cases`, see the [SPARK User's Guide¹⁹](#).

3.4 Debugging Failed Proof Attempts

GNATprove may report an error while verifying a program for any of the following reasons:

- there might be an error in the program; or
- the property may not be provable as written because more information is required; or
- the prover used by GNATprove may be unable to prove a perfectly valid property.

We spend the remainder of this section discussing the sometimes tricky task of debugging failed proof attempts.

¹⁹ https://docs.adacore.com/live/wave/spark2014/html/spark2014_ug/en/source/subprogram_contracts.html#contract-cases

3.4.1 Debugging Errors in Code or Specification

First, let's discuss the case where there's indeed an error in the program. There are two possibilities: the code may be incorrect or, equally likely, the specification may be incorrect. As an example, there's an error in our procedure `Incr_Until` below which makes its `Contract_Cases` unprovable.

Listing 11: `show_failed_proof_attempt.ads`

```

1 package Show_Failed_Proof_Attempt is
2
3   Incremented : Boolean := False;
4
5   procedure Incr_Until (X : in out Natural) with
6     Contract_Cases =>
7     (Incremented => X > X'Old,
8      others      => X = X'Old);
9
10 end Show_Failed_Proof_Attempt;
```

Listing 12: `show_failed_proof_attempt.adb`

```

1 package body Show_Failed_Proof_Attempt is
2
3   procedure Incr_Until (X : in out Natural) is
4     begin
5       if X < 1000 then
6         X := X + 1;
7         Incremented := True;
8       else
9         Incremented := False;
10      end if;
11    end Incr_Until;
12
13 end Show_Failed_Proof_Attempt;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Failed_Proof_Attempt_1
MD5: 814636ae9df6f4f66ad69f5099a5729b
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_failed_proof_attempt.ads:7:21: medium: contract case might fail
show_failed_proof_attempt.ads:8:21: medium: contract case might fail
gnatprove: unproved check messages considered as errors
```

Since this is an assertion that can be executed, it may help you find the problem if you run the program with assertions enabled on representative sets of inputs. This allows you to find bugs in both the code and its contracts. In this case, testing `Incr_Until` with an input greater than 1000 raises an exception at runtime.

Listing 13: `show_failed_proof_attempt.ads`

```

1 package Show_Failed_Proof_Attempt is
2
3   Incremented : Boolean := False;
4
5   procedure Incr_Until (X : in out Natural) with
```

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```
6     Contract_Cases =>
7       (Incremented => X > X'Old,
8        others      => X = X'Old);
9
10  end Show_Failed_Proof_Attempt;
```

Listing 14: show_failed_proof_attempt.adb

```
1  package body Show_Failed_Proof_Attempt is
2
3    procedure Incr_Until (X : in out Natural) is
4    begin
5      if X < 1000 then
6        X := X + 1;
7        Incremented := True;
8      else
9        Incremented := False;
10     end if;
11  end Incr_Until;
12
13  end Show_Failed_Proof_Attempt;
```

Listing 15: main.adb

```
1  with Show_Failed_Proof_Attempt; use Show_Failed_Proof_Attempt;
2
3  procedure Main is
4    Dummy : Integer;
5  begin
6    Dummy := 0;
7    Incr_Until (Dummy);
8
9    Dummy := 1000;
10   Incr_Until (Dummy);
11  end Main;
```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Failed_Proof_Attempt_2
MD5: bd87cb0f64a6468eaab3cad1678271db

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_failed_proof_attempt.ads:7:21: medium: contract case might fail
show_failed_proof_attempt.ads:8:21: medium: contract case might fail
gnatprove: unproved check messages considered as errors
```

Runtime output

```
raised ADA ASSERTIONS ASSERTION_ERROR : failed contract case at show_failed_proof_
↪ attempt.ads:8
```

The error message shows that the first contract case is failing, which means that `Incremented` is **True**. However, if we print the value of `Incremented` before returning, we see that it's **False**, as expected for the input we provided. The error here is that guards of contract cases are evaluated before the call, so our specification is wrong! To correct this, we should either write `X < 1000` as the guard of the first case or use a standard postcondition with an if-expression.

3.4.2 Debugging Cases where more Information is Required

Even if both the code and the assertions are correct, GNATprove may still report that it can't prove a verification condition for a property. This can happen for two reasons:

- The property may be unprovable because the code is missing some assertion. One category of these cases is due to the modularity of the analysis which, as we discussed above, means that GNATprove only knows about the properties of your subprograms that you have explicitly written.
- There may be some information missing in the logical model of the program used by GNATprove.

Let's look at the case where the code and the specification are correct but there's some information missing. As an example, GNATprove finds the postcondition of Increase to be unprovable.

Listing 16: show_failed_proof_attempt.ads

```

1 package Show_Failed_Proof_Attempt is
2
3   C : Natural := 100;
4
5   procedure Increase (X : in out Natural) with
6     Post => (if X'Old < C then X > X'Old else X = C);
7
8 end Show_Failed_Proof_Attempt;
```

Listing 17: show_failed_proof_attempt.adb

```

1 package body Show_Failed_Proof_Attempt is
2
3   procedure Increase (X : in out Natural) is
4     begin
5       if X < 90 then
6         X := X + 10;
7       elsif X >= C then
8         X := C;
9       else
10        X := X + 1;
11      end if;
12    end Increase;
13
14 end Show_Failed_Proof_Attempt;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Failed_Proof_Attempt_3
MD5: e01fc27a981bcb80757f30c94768237e
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_failed_proof_attempt.ads:6:49: medium: postcondition might fail, cannot prove
↳ X = C
gnatprove: unproved check messages considered as errors
```

This postcondition is a conditional. It says that if the parameter (X) is less than a certain value (C), its value will be increased by the procedure while if it's greater, its value will be set to C (saturated). When C has the value 100, the code of Increase adds 10 to the value of X if it was initially less than 90, increments X by 1 if it was between 90 and 99, and sets

X to 100 if it was greater or equal to 100. This behavior does satisfy the postcondition, so why is the postcondition not provable?

The values in the counterexample returned by GNATprove in its message gives us a clue: $C = 0$ and $X = 10$ and $X'Old = 0$. Indeed, if C is not equal to 100, our reasoning above is incorrect: the values of 0 for C and X on entry indeed result in X being 10 on exit, which violates the postcondition!

We probably didn't expect the value of C to change, or at least not to go below 90. But, in that case, we should have stated so by either declaring C to be constant or by adding a precondition to the Increase subprogram. If we do either of those, GNATprove is able to prove the postcondition.

3.4.3 Debugging Prover Limitations

Finally, there are cases where GNATprove provides a perfectly valid verification condition for a property, but it's nevertheless not proved by the automatic prover that runs in the later stages of the tool's execution. This is quite common. Indeed, GNATprove produces its verification conditions in first-order logic, which is not decidable, especially in combination with the rules of arithmetic. Sometimes, the automatic prover just needs more time. Other times, the prover will abandon the search almost immediately or loop forever without reaching a conclusive answer (either a proof or a counterexample).

For example, the postcondition of our GCD function below — which calculates the value of the GCD of two positive numbers using Euclide's algorithm — can't be verified with GNATprove's default settings.

Listing 18: show_failed_proof_attempt.ads

```
1 package Show_Failed_Proof_Attempt is
2
3   function GCD (A, B : Positive) return Positive with
4     Post =>
5       A mod GCD'Result = 0
6       and B mod GCD'Result = 0;
7
8 end Show_Failed_Proof_Attempt;
```

Listing 19: show_failed_proof_attempt.adb

```
1 package body Show_Failed_Proof_Attempt is
2
3   function GCD (A, B : Positive) return Positive is
4     begin
5       if A > B then
6         return GCD (A - B, B);
7       elsif B > A then
8         return GCD (A, B - A);
9       else
10        return A;
11      end if;
12    end GCD;
13
14 end Show_Failed_Proof_Attempt;
```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Failed_Proof_Attempt_4
MD5: a6f1a39ceb0793df8a00691d59a5d9ce

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_failed_proof_attempt.ads:5:08: medium: postcondition might fail, cannot prove
↳A mod GCD'Result = 0
gnatprove: unproved check messages considered as errors
```

The first thing we try is increasing the amount of time the prover is allowed to spend on each verification condition using the `--timeout` option of GNATprove (e.g., by using the dialog box in GNAT Studio). In this example, increasing it to one minute, which is relatively high, doesn't help. We can also specify an alternative automatic prover — if we have one — using the option `--prover` of GNATprove (or the dialog box). For our postcondition, we tried Alt-Ergo, cvc5, and Z3 without any luck.

Listing 20: show_failed_proof_attempt.ads

```
1 package Show_Failed_Proof_Attempt is
2
3     function GCD (A, B : Positive) return Positive with
4         Post =>
5             A mod GCD'Result = 0
6             and B mod GCD'Result = 0;
7
8 end Show_Failed_Proof_Attempt;
```

Listing 21: show_failed_proof_attempt.adb

```
1 package body Show_Failed_Proof_Attempt is
2
3     function GCD (A, B : Positive) return Positive
4     is
5         Result : Positive;
6     begin
7         if A > B then
8             Result := GCD (A - B, B);
9             pragma Assert ((A - B) mod Result = 0);
10            -- info: assertion proved
11            pragma Assert (B mod Result = 0);
12            -- info: assertion proved
13            pragma Assert (A mod Result = 0);
14            -- medium: assertion might fail
15        elsif B > A then
16            Result := GCD (A, B - A);
17            pragma Assert ((B - A) mod Result = 0);
18            -- info: assertion proved
19        else
20            Result := A;
21        end if;
22        return Result;
23    end GCD;
24
25 end Show_Failed_Proof_Attempt;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Failed_Proof_Attempt_5
MD5: 954ecbf2177705770c3a44a477c1de17
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
```

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```
show_failed_proof_attempt.adb:5:07: info: initialization of "Result" proved
show_failed_proof_attempt.adb:8:27: info: range check proved
show_failed_proof_attempt.adb:9:25: info: assertion proved
show_failed_proof_attempt.adb:9:33: info: division check proved
show_failed_proof_attempt.adb:11:25: info: assertion proved
show_failed_proof_attempt.adb:11:27: info: division check proved
show_failed_proof_attempt.adb:13:25: medium: assertion might fail [possible fix:
↳subprogram at show_failed_proof_attempt.ads:3 should mention A in a precondition]
show_failed_proof_attempt.adb:13:27: info: division check proved
show_failed_proof_attempt.adb:16:30: info: range check proved
show_failed_proof_attempt.adb:17:25: info: assertion proved
show_failed_proof_attempt.adb:17:33: info: division check proved
show_failed_proof_attempt.ads:5:10: info: division check proved
show_failed_proof_attempt.ads:6:12: medium: postcondition might fail, cannot prove
↳B mod GCD'Result = 0
show_failed_proof_attempt.ads:6:14: info: division check proved
gnatprove: unproved check messages considered as errors
```

To better understand the reason for the failure, we added intermediate assertions to simplify the proof and pin down the part that's causing the problem. Adding such assertions is often a good idea when trying to understand why a property is not proved. Here, provers can't verify that if both $A - B$ and B can be divided by `Result` so can A . This may seem surprising, but non-linear arithmetic, involving, for example, multiplication, modulo, or exponentiation, is a difficult topic for provers and is not handled very well in practice by any of the general-purpose ones like Alt-Ergo, cvc5, or Z3.

Note: For more details on how to investigate unproved checks, see the [SPARK User's Guide](#)²⁰.

3.5 Code Examples / Pitfalls

We end with some code examples and pitfalls.

3.5.1 Example #1

The package `Lists` defines a linked-list data structure. We call `Link(I,J)` to make a link from index I to index J and call `Goes_To(I,J)` to determine if we've created a link from index I to index J . The postcondition of `Link` uses `Goes_To` to state that there must be a link between its arguments once `Link` completes.

Listing 22: lists.ads

```
1 package Lists with SPARK_Mode is
2
3   type Index is new Integer;
4
5   function Goes_To (I, J : Index) return Boolean;
6
7   procedure Link (I, J : Index) with Post => Goes_To (I, J);
8
9 private
```

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²⁰ https://docs.adacore.com/live/wave/spark2014/html/spark2014_ug/en/source/how_to_investigate_unproved_checks.html

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```

10
11 type Cell (Is_Set : Boolean := True) is record
12     case Is_Set is
13         when True =>
14             Next : Index;
15         when False =>
16             null;
17     end case;
18 end record;
19
20 type Cell_Array is array (Index) of Cell;
21
22 Memory : Cell_Array;
23
24 end Lists;

```

Listing 23: lists.adb

```

1 package body Lists with SPARK_Mode is
2
3     function Goes_To (I, J : Index) return Boolean is
4     begin
5         if Memory (I).Is_Set then
6             return Memory (I).Next = J;
7         end if;
8         return False;
9     end Goes_To;
10
11     procedure Link (I, J : Index) is
12     begin
13         Memory (I) := (Is_Set => True, Next => J);
14     end Link;
15
16 end Lists;

```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Example_01
MD5: c2246948c584304d5694b49b4d1fd0fc

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
lists.ads:7:47: medium: postcondition might fail [possible fix: you should
↳consider adding a postcondition to function Goes_To or turning it into an
↳expression function]
gnatprove: unproved check messages considered as errors

```

This example is correct, but can't be verified by GNATprove. This is because Goes_To itself has no postcondition, so nothing is known about its result.

3.5.2 Example #2

We now redefine `Goes_To` as an expression function.

Listing 24: lists.ads

```
1 package Lists with SPARK_Mode is
2
3   type Index is new Integer;
4
5   function Goes_To (I, J : Index) return Boolean;
6
7   procedure Link (I, J : Index) with Post => Goes_To (I, J);
8
9 private
10
11  type Cell (Is_Set : Boolean := True) is record
12    case Is_Set is
13      when True =>
14        Next : Index;
15      when False =>
16        null;
17    end case;
18  end record;
19
20  type Cell_Array is array (Index) of Cell;
21
22  Memory : Cell_Array;
23
24  function Goes_To (I, J : Index) return Boolean is
25    (Memory (I).Is_Set and then Memory (I).Next = J);
26
27 end Lists;
```

Listing 25: lists.adb

```
1 package body Lists with SPARK_Mode is
2
3   procedure Link (I, J : Index) is
4     begin
5       Memory (I) := (Is_Set => True, Next => J);
6     end Link;
7
8 end Lists;
```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Example_02
MD5: c65953bbe8a5f9fb77a4d94e2dd875f9

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
lists.adb:5:18: info: discriminant check proved
lists.ads:7:47: info: postcondition proved
lists.ads:25:44: info: discriminant check proved
```

GNATprove can fully prove this version: `Goes_To` is an expression function, so its body is available for proof (specifically, for creating the postcondition needed for the proof).

3.5.3 Example #3

The package `Stacks` defines an abstract stack type with a `Push` procedure that adds an element at the top of the stack and a function `Peek` that returns the content of the element at the top of the stack (without removing it).

Listing 26: `stacks.ads`

```

1 package Stacks with SPARK_Mode is
2
3   type Stack is private;
4
5   function Peek (S : Stack) return Natural;
6   procedure Push (S : in out Stack; E : Natural) with
7     Post => Peek (S) = E;
8
9 private
10
11   Max : constant := 10;
12
13   type Stack_Array is array (1 .. Max) of Natural;
14
15   type Stack is record
16     Top      : Positive;
17     Content  : Stack_Array;
18   end record;
19
20   function Peek (S : Stack) return Natural is
21     (if S.Top in S.Content'Range then S.Content (S.Top) else 0);
22
23 end Stacks;
```

Listing 27: `stacks.adb`

```

1 package body Stacks with SPARK_Mode is
2
3   procedure Push (S : in out Stack; E : Natural) is
4   begin
5     if S.Top >= Max then
6       return;
7     end if;
8
9     S.Top := S.Top + 1;
10    S.Content (S.Top) := E;
11  end Push;
12
13 end Stacks;
```

Code block metadata

Project: `Courses.Intro_To_Spark.Proof_of_Program_Integrity.Example_03`
MD5: `917d624916c5ef14c4e454d6c56414fd`

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
stacks.ads:7:14: medium: postcondition might fail
gnatprove: unproved check messages considered as errors
```

This example isn't correct. The postcondition of `Push` is only satisfied if the stack isn't full when we call `Push`.

3.5.4 Example #4

We now change the behavior of Push so it raises an exception when the stack is full instead of returning.

Listing 28: stacks.ads

```
1 package Stacks with SPARK_Mode is
2
3   type Stack is private;
4
5   Is_Full_E : exception;
6
7   function Peek (S : Stack) return Natural;
8   procedure Push (S : in out Stack; E : Natural) with
9     Post => Peek (S) = E;
10
11 private
12
13   Max : constant := 10;
14
15   type Stack_Array is array (1 .. Max) of Natural;
16
17   type Stack is record
18     Top      : Positive;
19     Content  : Stack_Array;
20   end record;
21
22   function Peek (S : Stack) return Natural is
23     (if S.Top in S.Content'Range then S.Content (S.Top) else 0);
24
25 end Stacks;
```

Listing 29: stacks.adb

```
1 package body Stacks with SPARK_Mode is
2
3   procedure Push (S : in out Stack; E : Natural) is
4   begin
5     if S.Top >= Max then
6       raise Is_Full_E;
7     end if;
8
9     S.Top := S.Top + 1;
10    S.Content (S.Top) := E;
11  end Push;
12
13 end Stacks;
```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Example_04
MD5: b573ebe93f85ea171166b6953cbb8956

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
stacks.adb:6:10: medium: exception might be raised
gnatprove: unproved check messages considered as errors
```

The postcondition of Push is now proved because GNATprove only considers execution paths

leading to normal termination. But it issues a message warning that exception `Is_Full_E` may be raised at runtime.

3.5.5 Example #5

Let's add a precondition to `Push` stating that the stack shouldn't be full.

Listing 30: stacks.ads

```

1 package Stacks with SPARK_Mode is
2
3   type Stack is private;
4
5   Is_Full_E : exception;
6
7   function Peek (S : Stack) return Natural;
8   function Is_Full (S : Stack) return Boolean;
9   procedure Push (S : in out Stack; E : Natural) with
10      Pre => not Is_Full (S),
11      Post => Peek (S) = E;
12
13 private
14
15   Max : constant := 10;
16
17   type Stack_Array is array (1 .. Max) of Natural;
18
19   type Stack is record
20     Top      : Positive;
21     Content : Stack_Array;
22   end record;
23
24   function Peek (S : Stack) return Natural is
25     (if S.Top in S.Content'Range then S.Content (S.Top) else 0);
26   function Is_Full (S : Stack) return Boolean is (S.Top >= Max);
27
28 end Stacks;
```

Listing 31: stacks.adb

```

1 package body Stacks with SPARK_Mode is
2
3   procedure Push (S : in out Stack; E : Natural) is
4     begin
5       if S.Top >= Max then
6         raise Is_Full_E;
7       end if;
8       S.Top := S.Top + 1;
9       S.Content (S.Top) := E;
10    end Push;
11
12 end Stacks;
```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Example_05
MD5: 63c2dfd68dd5acd91d8d497206e7423e

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
stacks.adb:6:10: info: raise statement or expression proved unreachable
stacks.adb:8:22: info: overflow check proved
stacks.adb:9:19: info: index check proved
stacks.ads:11:14: info: postcondition proved
stacks.ads:25:52: info: index check proved
```

This example is correct. With the addition of the precondition, GNATprove can now verify that `Is_Full_E` can never be raised at runtime.

3.5.6 Example #6

The package `Memories` defines a type `Chunk` that models chunks of memory. Each element of the array, represented by its index, corresponds to one data element. The procedure `Read_Record` reads two pieces of data starting at index `From` out of the chunk represented by the value of `Memory`.

Listing 32: `memories.ads`

```
1 package Memories is
2
3   type Chunk is array (Integer range <>) of Integer
4     with Predicate => Chunk'Length >= 10;
5
6   function Is_Too_Coarse (V : Integer) return Boolean;
7
8   procedure Treat_Value (V : out Integer);
9
10 end Memories;
```

Listing 33: `read_record.adb`

```
1 with Memories; use Memories;
2
3 procedure Read_Record (Memory : Chunk; From : Integer)
4   with SPARK_Mode => On,
5     Pre => From in Memory'First .. Memory'Last - 2
6 is
7   function Read_One (First : Integer; Offset : Integer) return Integer
8     with Pre => First + Offset in Memory'Range
9   is
10    Value : Integer := Memory (First + Offset);
11    begin
12      if Is_Too_Coarse (Value) then
13        Treat_Value (Value);
14      end if;
15      return Value;
16    end Read_One;
17
18    Data1, Data2 : Integer;
19
20  begin
21    Data1 := Read_One (From, 1);
22    Data2 := Read_One (From, 2);
23  end Read_Record;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Example_06
MD5: aec8014dc291708999092fa123ee7416
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
read_record.adb:8:24: medium: overflow check might fail, cannot prove lower bound
↳for First + Offset [reason for check: result of addition must fit in a 32-bit
↳machine integer] [possible fix: use pragma Overflow_Mode or switch -gnatol3 or
↳unit Ada.Numerics.Big_Numerics.Big_Integers]
gnatprove: unproved check messages considered as errors
```

This example is correct, but it can't be verified by GNATprove, which analyses `Read_One` on its own and notices that an overflow may occur in its precondition in certain contexts.

3.5.7 Example #7

Let's rewrite the precondition of `Read_One` to avoid any possible overflow.

Listing 34: memories.ads

```
1 package Memories is
2
3   type Chunk is array (Integer range <>) of Integer
4     with Predicate => Chunk'Length >= 10;
5
6   function Is_Too_Coarse (V : Integer) return Boolean;
7
8   procedure Treat_Value (V : out Integer);
9
10 end Memories;
```

Listing 35: read_record.adb

```
1 with Memories; use Memories;
2
3 procedure Read_Record (Memory : Chunk; From : Integer)
4   with SPARK_Mode => On,
5     Pre => From in Memory'First .. Memory'Last - 2
6 is
7   function Read_One (First : Integer; Offset : Integer) return Integer
8     with Pre => First >= Memory'First
9     and then Offset in 0 .. Memory'Last - First
10  is
11    Value : Integer := Memory (First + Offset);
12  begin
13    if Is_Too_Coarse (Value) then
14      Treat_Value (Value);
15    end if;
16    return Value;
17  end Read_One;
18
19  Data1, Data2 : Integer;
20
21 begin
22   Data1 := Read_One (From, 1);
23   Data2 := Read_One (From, 2);
24 end Read_Record;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Example_07
MD5: 6b4c6a41b652ad76bc7ef8934dcd9bfc
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
read_record.adb:9:49: medium: overflow check might fail, cannot prove lower bound_
↳for Memory'Last - First [reason for check: result of subtraction must fit in a_
↳32-bits machine integer] [possible fix: use pragma Overflow_Mode or switch -
↳gnatol3 or unit Ada.Numerics.Big_Numerics.Big_Integers]
gnatprove: unproved check messages considered as errors
```

This example is also not correct: unfortunately, our attempt to correct `Read_One`'s precondition failed. For example, an overflow will occur at runtime if `First` is **Integer'Last** and `Memory'Last` is negative. This is possible here because type `Chunk` uses **Integer** as base index type instead of **Natural** or **Positive**.

3.5.8 Example #8

Let's completely remove the precondition of `Read_One`.

Listing 36: memories.ads

```
1 package Memories is
2
3     type Chunk is array (Integer range <>) of Integer
4       with Predicate => Chunk'Length >= 10;
5
6     function Is_Too_Coarse (V : Integer) return Boolean;
7
8     procedure Treat_Value (V : out Integer);
9
10 end Memories;
```

Listing 37: read_record.adb

```
1 with Memories; use Memories;
2
3 procedure Read_Record (Memory : Chunk; From : Integer)
4   with SPARK_Mode => On,
5     Pre => From in Memory'First .. Memory'Last - 2
6 is
7   function Read_One (First : Integer; Offset : Integer) return Integer is
8     Value : Integer := Memory (First + Offset);
9   begin
10    if Is_Too_Coarse (Value) then
11      Treat_Value (Value);
12    end if;
13    return Value;
14  end Read_One;
15
16  Data1, Data2 : Integer;
17
18 begin
19   Data1 := Read_One (From, 1);
20   Data2 := Read_One (From, 2);
21 end Read_Record;
```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Example_08
 MD5: 5a806fb84b50d2dc1f2af428b1bc8d0a

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
read_record.adb:5:51: info: overflow check proved
read_record.adb:8:40: info: overflow check proved, in call inlined at read_record.
↪adb:19
read_record.adb:8:40: info: index check proved, in call inlined at read_record.
↪adb:19
read_record.adb:8:40: info: overflow check proved, in call inlined at read_record.
↪adb:20
read_record.adb:8:40: info: index check proved, in call inlined at read_record.
↪adb:20
```

This example is correct and fully proved. We could have fixed the contract of `Read_One` to correctly handle both positive and negative values of `Memory`' `Last`, but we found it simpler to let the function be inlined for proof by removing its precondition.

3.5.9 Example #9

The procedure `Compute` performs various computations on its argument. The computation performed depends on its input range and is reflected in its contract, which we express using a `Contract_Cases` aspect.

Listing 38: compute.adb

```
1 procedure Compute (X : in out Integer) with
2   Contract_Cases => ((X in -100 .. 100) => X = X'Old * 2,
3                     (X in 0 .. 199) => X = X'Old + 1,
4                     (X in -199 .. 0) => X = X'Old - 1,
5                     X >= 200 => X = 200,
6                     others => X = -200)
7 is
8 begin
9   if X in -100 .. 100 then
10    X := X * 2;
11  elsif X in 0 .. 199 then
12    X := X + 1;
13  elsif X in -199 .. 0 then
14    X := X - 1;
15  elsif X >= 200 then
16    X := 200;
17  else
18    X := -200;
19  end if;
20 end Compute;
```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Example_09
 MD5: 51962d1bb6dd1b081ed498dd11559685

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
compute.adb:2:03: medium: contract cases might not be disjoint
compute.adb:3:41: medium: contract case might fail
compute.adb:4:41: medium: contract case might fail
gnatprove: unproved check messages considered as errors
```

This example isn't correct. We duplicated the content of `Compute`'s body in its contract. This is incorrect because the semantics of `Contract_Cases` require disjoint cases, just like a case statement. The counterexample returned by GNATprove shows that $X = 0$ is covered by two different case-guards (the first and the second).

3.5.10 Example #10

Let's rewrite the contract of `Compute` to avoid overlapping cases.

Listing 39: `compute.adb`

```
1 procedure Compute (X : in out Integer) with
2   Contract_Cases => ((X in 0 .. 199) => X >= X'Old,
3                     (X in -199 .. -1) => X <= X'Old,
4                     X >= 200      => X = 200,
5                     X < -200     => X = -200)
6 is
7 begin
8   if X in -100 .. 100 then
9     X := X * 2;
10  elsif X in 0 .. 199 then
11    X := X + 1;
12  elsif X in -199 .. 0 then
13    X := X - 1;
14  elsif X >= 200 then
15    X := 200;
16  else
17    X := -200;
18  end if;
19 end Compute;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Example_10
MD5: 01d33b10fd60f384ffa4ae8feale7d87
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
compute.adb:2:03: medium: contract cases might not be complete
gnatprove: unproved check messages considered as errors
```

This example is still not correct. GNATprove can successfully prove the different cases are disjoint and also successfully verify each case individually. This isn't enough, though: a `Contract_Cases` must cover all cases. Here, we forgot the value `-200`, which is what GNATprove reports in its counterexample.

STATE ABSTRACTION

Abstraction is a key concept in programming that can drastically simplify both the implementation and maintenance of code. It's particularly well suited to SPARK and its modular analysis. This section explains what state abstraction is and how you use it in SPARK. We explain how it impacts GNATprove's analysis both in terms of information flow and proof of program properties.

State abstraction allows us to:

- express dependencies that wouldn't otherwise be expressible because some data that's read or written isn't visible at the point where a subprogram is declared — examples are dependencies on data, for which we use the `Global` contract, and on flow, for which we use the `Depends` contract.
- reduce the number of variables that need to be considered in flow analysis and proof, a reduction which may be critical in order to scale the analysis to programs with thousands of global variables.

4.1 What's an Abstraction?

Abstraction is an important part of programming language design. It provides two views of the same object: an abstract one and a refined one. The abstract one — usually called *specification* — describes what the object does in a coarse way. A subprogram's specification usually describes how it should be called (e.g., parameter information such as how many and of what types) as well as what it does (e.g., returns a result or modifies one or more of its parameters).

Contract-based programming, as supported in Ada, allows contracts to be added to a subprogram's specification. You use contracts to describe the subprogram's behavior in a more fine-grained manner, but all the details of how the subprogram actually works are left to its refined view, its implementation.

Take a look at the example code shown below.

Listing 1: increase.ads

```
1 procedure Increase (X : in out Integer) with
2   Global => null,
3   Pre    => X <= 100,
4   Post   => X'Old < X;
```

Listing 2: increase.adb

```
1 procedure Increase (X : in out Integer) is
2   begin
3     X := X + 1;
4   end Increase;
```


Code block metadata

```
Project: Courses.Intro_To_Spark.State_Abstraction.No_Abstraction
MD5: c4c8f229aeb1b5c12744d26369a8603f
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
increase.adb:3:11: info: overflow check proved
increase.ads:2:03: info: data dependencies proved
increase.ads:4:13: info: postcondition proved
```

We've written a specification of the subprogram `Increase` to say that it's called with a single argument, a variable of type **Integer** whose initial value is less than 100. Our contract says that the only effect of the subprogram is to increase the value of its argument.

4.2 Why is Abstraction Useful?

A good abstraction of a subprogram's implementation is one whose specification precisely and completely summarizes what its callers can rely on. In other words, a caller of that subprogram shouldn't rely on any behavior of its implementation if that behavior isn't documented in its specification.

For example, callers of the subprogram `Increase` can assume that it always strictly increases the value of its argument. In the code snippet shown below, this means the loop must terminate.

Listing 3: `increase.ads`

```
1 procedure Increase (X : in out Integer) with
2   Global => null,
3   Pre    => X <= 100,
4   Post   => X'Old < X;
```

Listing 4: `client.adb`

```
1 with Increase;
2 procedure Client is
3   X : Integer := 0;
4 begin
5   while X <= 100 loop      -- The loop will terminate
6     Increase (X);         -- Increase can be called safely
7   end loop;
8   pragma Assert (X = 101); -- Will this hold?
9 end Client;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.State_Abstraction.Using_Abstraction
MD5: 9cd07cb04ae2194343931f0561693be4
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
client.adb:8:19: medium: assertion might fail
gnatprove: unproved check messages considered as errors
```

Callers can also assume that the implementation of `Increase` won't cause any runtime errors when called in the loop. On the other hand, nothing in the specification guarantees that the assertion shown above is correct: it may fail if `Increase`'s implementation is changed.

If you follow this basic principle, abstraction can bring you significant benefits. It simplifies both your program's implementation and verification. It also makes maintenance and code reuse much easier since changes to the implementation of an object shouldn't affect the code using this object. Your goal in using it is that it should be enough to understand the specification of an object in order to use that object, since understanding the specification is usually much simpler than understanding the implementation.

GNATprove relies on the abstraction defined by subprogram contracts and therefore doesn't prove the assertion after the loop in `Client` above.

4.3 Abstraction of a Package's State

Subprograms aren't the only objects that benefit from abstraction. The state of a package — the set of persistent variables defined in it — can also be hidden from external users. You achieve this form of abstraction — called *state abstraction* — by defining variables in the body or private part of a package so they can only be accessed through subprogram calls. For example, our `Stack` package shown below provides an abstraction for a `Stack` object which can only be modified using the `Pop` and `Push` procedures.

```
package Stack is
  procedure Pop (E : out Element);
  procedure Push (E : in Element);
end Stack;

package body Stack is
  Content : Element_Array (1 .. Max);
  Top      : Natural;
  ...
end Stack;
```

The fact that we implemented it using an array is irrelevant to the caller. We could change that without impacting our callers' code.

4.4 Declaring a State Abstraction

Hidden state influences a program's behavior, so SPARK allows that state to be declared. You can use the `Abstract_State` aspect, an abstraction that names a state, to do this, but you aren't required to use it even for a package with hidden state. You can use several state abstractions to declare the hidden state of a single package or you can use it for a package with no hidden state at all. However, since SPARK doesn't allow aliasing, different state abstractions must always refer to disjoint sets of variables. A state abstraction isn't a variable: it doesn't have a type and can't be used inside expressions, either those in bodies or contracts.

As an example of the use of this aspect, we can optionally define a state abstraction for the entire hidden state of the `Stack` package like this:

```
package Stack with
  Abstract_State => The_Stack
is
  ...
```

Alternatively, we can define a state abstraction for each hidden variable:

```
package Stack with
  Abstract_State => (Top_State, Content_State)
is
  ...
```

Remember: a state abstraction isn't a variable (it has no type) and can't be used inside expressions. For example:

```
pragma Assert (Stack.Top_State = ...);
-- compilation error: Top_State is not a variable
```

4.5 Refining an Abstract State

Once you've declared an abstract state in a package, you must refine it into its constituents using a `Refined_State` aspect. You must place the `Refined_State` aspect on the package body even if the package wouldn't otherwise have required a body. For each state abstraction you've declared for the package, you list the set of variables represented by that state abstraction in its refined state.

If you specify an abstract state for a package, it must be complete, meaning you must have listed every hidden variable as part of some state abstraction. For example, we must add a `Refined_State` aspect on our `Stack` package's body linking the state abstraction (`The_Stack`) to the entire hidden state of the package, which consists of both `Content` and `Top`.

Listing 5: stack.ads

```
1 package Stack with
2   Abstract_State => The_Stack
3 is
4   type Element is new Integer;
5
6   procedure Pop (E : out Element);
7   procedure Push (E : Element);
8
9 end Stack;
```

Listing 6: stack.adb

```
1 package body Stack with
2   Refined_State => (The_Stack => (Content, Top))
3 is
4   Max : constant := 100;
5
6   type Element_Array is array (1 .. Max) of Element;
7
8   Content : Element_Array := (others => 0);
9   Top      : Natural range 0 .. Max := 0;
10  -- Both Content and Top must be listed in the list of
11  -- constituents of The_Stack
12
13  procedure Pop (E : out Element) is
14  begin
15    E := Content (Top);
16    Top := Top - 1;
17  end Pop;
18
19  procedure Push (E : Element) is
```

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```

20   begin
21     Top           := Top + 1;
22     Content (Top) := E;
23   end Push;
24
25 end Stack;

```

Code block metadata

```

Project: Courses.Intro_To_Spark.State_Abstraction.Refined_State
MD5: 3a794c7a4e4920dab7d01248e50901ab

```

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
stack.ads:6:20: info: initialization of "E" proved

```

4.6 Representing Private Variables

You can refine state abstractions in the package body, where all the variables are visible. When only the package's specification is available, you need a way to specify which state abstraction each private variable belongs to. You do this by adding the `Part_Of` aspect to the variable's declaration.

`Part_Of` annotations are mandatory: if you gave a package an abstract state annotation, you must link all the hidden variables defined in its private part to a state abstraction. For example:

Listing 7: stack.ads

```

1 package Stack with
2   Abstract_State => The_Stack
3 is
4   type Element is new Integer;
5
6   procedure Pop (E : out Element);
7   procedure Push (E : Element);
8
9 private
10
11   Max : constant := 100;
12
13   type Element_Array is array (1 .. Max) of Element;
14
15   Content : Element_Array with Part_Of => The_Stack;
16   Top      : Natural range 0 .. Max with Part_Of => The_Stack;
17
18 end Stack;

```

Code block metadata

```

Project: Courses.Intro_To_Spark.State_Abstraction.Private_Variables
MD5: 3b5f7edca8a4511071d2397197b01fda

```

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...

Since we chose to define Content and Top in Stack's private part instead of its body, we had to add a Part_Of aspect to both of their declarations, associating them with the state abstraction The_Stack, even though it's the only state abstraction. However, we still need to list them in the Refined_State aspect in Stack's body.

```
package body Stack with
  Refined_State => (The_Stack => (Content, Top))
```

4.7 Additional State

4.7.1 Nested Packages

So far, we've only discussed hidden variables. But variables aren't the only component of a package's state. If a package P contains a nested package, the nested package's state is also part of P's state. If the nested package is hidden, its state is part of P's hidden state and must be listed in P's state refinement.

We see this in the example below, where the package Hidden_Nested's hidden state is part of P's hidden state.

Listing 8: p.ads

```
1 package P with
2   Abstract_State => State
3 is
4   package Visible_Nested with
5     Abstract_State => Visible_State
6   is
7     procedure Get (E : out Integer);
8   end Visible_Nested;
9 end P;
```

Listing 9: p.adb

```
1 package body P with
2   Refined_State => (State => Hidden_Nested.Hidden_State)
3 is
4   package Hidden_Nested with
5     Abstract_State => Hidden_State,
6     Initializes    => Hidden_State
7   is
8     function Get return Integer;
9   end Hidden_Nested;
10
11   package body Hidden_Nested with
12     Refined_State => (Hidden_State => Cnt)
13   is
14     Cnt : Integer := 0;
15
16     function Get return Integer is (Cnt);
17   end Hidden_Nested;
18
19   package body Visible_Nested with
20     Refined_State => (Visible_State => Checked)
```

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```

21  is
22    Checked : Boolean := False;
23
24    procedure Get (E : out Integer) is
25    begin
26      Checked := True;
27      E := Hidden_Nested.Get;
28    end Get;
29  end Visible_Nested;
30 end P;

```

Code block metadata

```

Project: Courses.Intro_To_Spark.State_Abstraction.Nested_Packages
MD5: 8260089cbd651de296dd790506c76fd8

```

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
p.adb:6:07: info: flow dependencies proved
p.ads:7:22: info: initialization of "E" proved

```

Any visible state of `Hidden_Nested` would also have been part of `P`'s hidden state. However, if `P` contains a visible nested package, that nested package's state isn't part of `P`'s hidden state. Instead, you should declare that package's hidden state in a separate state abstraction on its own declaration, like we did above for `Visible_Nested`.

4.7.2 Constants that Depend on Variables

Some constants are also possible components of a state abstraction. These are constants whose value depends either on a variable or a subprogram parameter. They're handled as variables during flow analysis because they participate in the flow of information between variables throughout the program. Therefore, GNATprove considers these constants to be part of a package's state just like it does for variables.

If you've specified a state abstraction for a package, you must list such hidden constants declared in that package in the state abstraction refinement. However, constants that don't depend on variables don't participate in the flow of information and must not appear in a state refinement.

Let's look at this example.

Listing 10: stack.ads

```

1  package Stack with
2    Abstract_State => The_Stack
3  is
4    type Element is new Integer;
5
6    procedure Pop (E : out Element);
7    procedure Push (E : Element);
8  end Stack;

```

Listing 11: configuration.ads

```

1  package Configuration with
2    Initializes => External_Variable
3  is

```

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```
4   External_Variable : Positive with Volatile;
5 end Configuration;
```

Listing 12: stack.adb

```
1 with Configuration;
2 pragma Elaborate (Configuration);
3
4 package body Stack with
5   Refined_State => (The_Stack => (Content, Top, Max))
6   -- Max has variable inputs. It must appear as a
7   -- constituent of The_Stack
8 is
9   Max : constant Positive := Configuration.External_Variable;
10
11   type Element_Array is array (1 .. Max) of Element;
12
13   Content : Element_Array := (others => 0);
14   Top      : Natural range 0 .. Max := 0;
15
16   procedure Pop (E : out Element) is
17   begin
18     E := Content (Top);
19     Top := Top - 1;
20   end Pop;
21
22   procedure Push (E : Element) is
23   begin
24     Top := Top + 1;
25     Content (Top) := E;
26   end Push;
27
28 end Stack;
```

Code block metadata

Project: Courses.Intro_To_Spark.State_Abstraction.Constants_And_Variables
MD5: 109a6340ef0f3b0dc88e0fe5888b9a53

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
stack.ads:6:20: info: initialization of "E" proved
configuration.ads:2:03: info: flow dependencies proved
```

Here, Max — the maximum number of elements that can be stored in the stack — is initialized from a variable in an external package. Because of this, we must include Max as part of the state abstraction The_Stack.

Note: For more details on state abstractions, see the [SPARK User's Guide](#)²¹.

²¹ https://docs.adacore.com/live/wave/spark2014/html/spark2014_ug/en/source/package_contracts.html#state-abstraction

4.8 Subprogram Contracts

4.8.1 Global and Depends

Hidden variables can only be accessed through subprogram calls, so you document how state abstractions are modified during the program's execution via the contracts of those subprograms. You use `Global` and `Depends` contracts to specify which of the state abstractions are used by a subprogram and how values flow through the different variables. The `Global` and `Depends` contracts that you write when referring to state abstractions are often less precise than contracts referring to visible variables since the possibly different dependencies of the hidden variables contained within a state abstraction are collapsed into a single dependency.

Let's add `Global` and `Depends` contracts to the `Pop` procedure in our stack.

Listing 13: stack.ads

```

1 package Stack with
2   Abstract_State => (Top_State, Content_State)
3 is
4   type Element is new Integer;
5
6   procedure Pop (E : out Element) with
7     Global => (Input => Content_State,
8               In_Out => Top_State),
9     Depends => (Top_State => Top_State,
10              E => (Content_State, Top_State));
11
12 end Stack;
```

Code block metadata

Project: Courses.Intro_To_Spark.State_Abstraction.Global_Depends
MD5: a7b383c35508d6a8294bf7cf0fe332ac

Prover output

Phase 1 of 2: generation of `Global` contracts ...
Phase 2 of 2: analysis of data and information flow ...

In this example, the `Pop` procedure only modifies the value of the hidden variable `Top`, while `Content` is unchanged. By using distinct state abstractions for the two variables, we're able to preserve this semantic in the contract.

Let's contrast this example with a different representation of `Global` and `Depends` contracts, this time using a single abstract state.

Listing 14: stack.ads

```

1 package Stack with
2   Abstract_State => The_Stack
3 is
4   type Element is new Integer;
5
6   procedure Pop (E : out Element) with
7     Global => (In_Out => The_Stack),
8     Depends => ((The_Stack, E) => The_Stack);
9
10 end Stack;
```

Code block metadata

Project: Courses.Intro_To_Spark.State_Abstraction.Global_Single_Abstract_State
MD5: f89f6026fa5ee3c18baf0af9d7c3dbca

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...

Here, `Top_State` and `Content_State` are merged into a single state abstraction, `The_Stack`. By doing so, we've hidden the fact that `Content` isn't modified (though we're still showing that `Top` may be modified). This loss in precision is reasonable here, since it's the whole point of the abstraction. However, you must be careful not to aggregate unrelated hidden state because this risks their annotations becoming meaningless.

Even though imprecise contracts that consider state abstractions as a whole are perfectly reasonable for users of a package, you should write `Global` and `Depends` contracts that are as precise as possible within the package body. To allow this, SPARK introduces the notion of *refined contracts*, which are precise contracts specified on the bodies of subprograms where state refinements are visible. These contracts are the same as normal `Global` and `Depends` contracts except they refer directly to the hidden state of the package.

When a subprogram is called inside the package body, you should write refined contracts instead of the general ones so that the verification can be as precise as possible. However, refined `Global` and `Depends` are optional: if you don't specify them, GNATprove will compute them to check the package's implementation.

For our `Stack` example, we could add refined contracts as shown below.

Listing 15: stack.ads

```
1 package Stack with
2   Abstract_State => The_Stack
3 is
4   type Element is new Integer;
5
6   procedure Pop (E : out Element) with
7     Global => (In_Out => The_Stack),
8     Depends => ((The_Stack, E) => The_Stack);
9
10  procedure Push (E : Element) with
11    Global => (In_Out => The_Stack),
12    Depends => (The_Stack => (The_Stack, E));
13
14 end Stack;
```

Listing 16: stack.adb

```
1 package body Stack with
2   Refined_State => (The_Stack => (Content, Top))
3 is
4   Max : constant := 100;
5
6   type Element_Array is array (1 .. Max) of Element;
7
8   Content : Element_Array := (others => 0);
9   Top : Natural range 0 .. Max := 0;
10
11  procedure Pop (E : out Element) with
12    Refined_Global => (Input => Content,
13                      In_Out => Top),
14    Refined_Depends => (Top => Top,
```

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```

15         E => (Content, Top))
16     is
17     begin
18         E := Content (Top);
19         Top := Top - 1;
20     end Pop;
21
22     procedure Push (E : Element) with
23         Refined_Global => (In_Out => (Content, Top)),
24         Refined_Depends => (Content =>+ (Content, Top, E),
25                             Top      => Top) is
26     begin
27         Top := Top + 1;
28         Content (Top) := E;
29     end Push;
30
31 end Stack;

```

Code block metadata

Project: Courses.Intro_To_Spark.State_Abstraction.Global_Refined
MD5: b7e700645885155ea7faf2f4170f0462

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...

4.8.2 Preconditions and Postconditions

We mostly express functional properties of subprograms using preconditions and postconditions. These are standard Boolean expressions, so they can't directly refer to state abstractions. To work around this restriction, we can define functions to query the value of hidden variables. We then use these functions in place of the state abstraction in the contract of other subprograms.

For example, we can query the state of the stack with functions `Is_Empty` and `Is_Full` and call these in the contracts of procedures `Pop` and `Push`:

Listing 17: stack.ads

```

1 package Stack is
2     type Element is new Integer;
3
4     function Is_Empty return Boolean;
5     function Is_Full  return Boolean;
6
7     procedure Pop (E : out Element) with
8         Pre => not Is_Empty,
9         Post => not Is_Full;
10
11    procedure Push (E : Element) with
12        Pre => not Is_Full,
13        Post => not Is_Empty;
14
15 end Stack;

```

Listing 18: stack.adb

```
1 package body Stack is
2
3   Max : constant := 100;
4
5   type Element_Array is array (1 .. Max) of Element;
6
7   Content : Element_Array := (others => 0);
8   Top      : Natural range 0 .. Max := 0;
9
10  function Is_Empty return Boolean is (Top = 0);
11  function Is_Full  return Boolean is (Top = Max);
12
13  procedure Pop (E : out Element) is
14  begin
15      E := Content (Top);
16      Top := Top - 1;
17  end Pop;
18
19  procedure Push (E : Element) is
20  begin
21      Top := Top + 1;
22      Content (Top) := E;
23  end Push;
24
25 end Stack;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.State_Abstraction.Pre_Postconditions_1
MD5: fe9d4b65ba1beeabc7cf0feda29b8b3c
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
stack.adb:15:23: info: index check proved
stack.adb:16:18: info: range check proved
stack.adb:21:28: info: range check proved
stack.adb:22:16: info: index check proved
stack.ads:7:19: info: initialization of "E" proved
stack.ads:9:14: info: postcondition proved
stack.ads:13:14: info: postcondition proved
```

Just like we saw for Global and Depends contracts, you may often find it useful to have a more precise view of functional contracts in the context where the hidden variables are visible. You do this using expression functions in the same way we did for the functions `Is_Empty` and `Is_Full` above. As expression function, bodies act as contracts for GNATprove, so they automatically give a more precise version of the contracts when their implementation is visible.

You may often need a more constraining contract to verify the package's implementation but want to be less strict outside the abstraction. You do this using the `Refined_Post` aspect. This aspect, when placed on a subprogram's body, provides stronger guarantees to internal callers of a subprogram. If you provide one, the refined postcondition must imply the subprogram's postcondition. This is checked by GNATprove, which reports a failing postcondition if the refined postcondition is too weak, even if it's actually implied by the subprogram's body. SPARK doesn't perform a similar verification for normal preconditions.

For example, we can refine the postconditions in the bodies of `Pop` and `Push` to be more detailed than what we wrote for them in their specification.

Listing 19: stack.ads

```

1 package Stack is
2   type Element is new Integer;
3
4   function Is_Empty return Boolean;
5   function Is_Full  return Boolean;
6
7   procedure Pop (E : out Element) with
8     Pre => not Is_Empty,
9     Post => not Is_Full;
10
11  procedure Push (E : Element) with
12    Pre => not Is_Full,
13    Post => not Is_Empty;
14
15 end Stack;
```

Listing 20: stack.adb

```

1 package body Stack is
2
3   Max : constant := 100;
4
5   type Element_Array is array (1 .. Max) of Element;
6
7   Content : Element_Array := (others => 0);
8   Top      : Natural range 0 .. Max := 0;
9
10  function Is_Empty return Boolean is (Top = 0);
11  function Is_Full  return Boolean is (Top = Max);
12
13  procedure Pop (E : out Element) with
14    Refined_Post => not Is_Full and E = Content (Top)'Old
15  is
16  begin
17    E := Content (Top);
18    Top := Top - 1;
19  end Pop;
20
21  procedure Push (E : Element) with
22    Refined_Post => not Is_Empty and E = Content (Top)
23  is
24  begin
25    Top := Top + 1;
26    Content (Top) := E;
27  end Push;
28
29 end Stack;
```

Code block metadata

Project: Courses.Intro_To_Spark.State_Abstraction.Pre_Postconditions_2
MD5: 4691565d58ba039b3cbd06e65cecfa88

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
stack.adb:14:22: info: refined post proved
stack.adb:14:51: info: index check proved

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```
stack.adb:17:23: info: index check proved
stack.adb:18:18: info: range check proved
stack.adb:22:22: info: refined post proved
stack.adb:22:52: info: index check proved
stack.adb:25:28: info: range check proved
stack.adb:26:16: info: index check proved
stack.ads:7:19: info: initialization of "E" proved
stack.ads:9:14: info: postcondition proved
stack.ads:13:14: info: postcondition proved
```

Note: For more details on refinement in contracts, see the [SPARK User's Guide](#)²².

4.9 Initialization of Local Variables

As part of flow analysis, GNATprove checks for the proper initialization of variables. Therefore, flow analysis needs to know which variables are initialized during the package's elaboration.

You can use the `Initializes` aspect to specify the set of visible variables and state abstractions that are initialized during the elaboration of a package. An `Initializes` aspect can't refer to a variable that isn't defined in the unit since, in SPARK, a package can only initialize variables declared immediately within the package.

`Initializes` aspects are optional. If you don't supply any, they'll be derived by GNATprove.

For our `Stack` example, we could add an `Initializes` aspect.

Listing 21: `stack.ads`

```
1 package Stack with
2   Abstract_State => The_Stack,
3   Initializes   => The_Stack
4 is
5   type Element is new Integer;
6
7   procedure Pop (E : out Element);
8
9 end Stack;
```

Listing 22: `stack.adb`

```
1 package body Stack with
2   Refined_State => (The_Stack => (Content, Top))
3 is
4   Max : constant := 100;
5
6   type Element_Array is array (1 .. Max) of Element;
7
8   Content : Element_Array := (others => 0);
9   Top      : Natural range 0 .. Max := 0;
10
11  procedure Pop (E : out Element) is
12  begin
13    E := Content (Top);
```

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²² https://docs.adacore.com/live/wave/spark2014/html/spark2014_ug/en/source/subprogram_contracts.html#state-abstraction-and-contracts

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```

14     Top := Top - 1;
15     end Pop;
16
17 end Stack;

```

Code block metadata

```

Project: Courses.Intro_To_Spark.State_Abstraction.Local_Init
MD5: 710e74959fd2ef8f5089c4636d7ec13b

```

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
stack.ads:3:03: info: flow dependencies proved
stack.ads:7:20: info: initialization of "E" proved

```

Flow analysis also checks for dependencies between variables, so it must be aware of how information flows through the code that performs the initialization of states. We discussed one use of the `Initializes` aspect above. But you also can use it to provide flow information. If the initial value of a variable or state abstraction is dependent on the value of another visible variable or state abstraction from another package, you must list this dependency in the `Initializes` contract. You specify the list of entities on which a variable's initial value depends using an arrow following that variable's name.

Let's look at this example:

Listing 23: q.ads

```

1 package Q is
2   External_Variable : Integer := 2;
3 end Q;

```

Listing 24: p.ads

```

1 with Q;
2 package P with
3   Initializes => (V1, V2 => Q.External_Variable)
4 is
5   V1 : Integer := 0;
6   V2 : Integer := Q.External_Variable;
7 end P;

```

Code block metadata

```

Project: Courses.Intro_To_Spark.State_Abstraction.Initializes
MD5: c8aa7f21729f3b926bf3d25a826cccb2

```

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
p.ads:3:03: info: flow dependencies proved

```

Here we indicated that `V2`'s initial value depends on the value of `Q.External_Variable` by including that dependency in the `Initializes` aspect of `P`. We didn't list any dependency for `V1` because its initial value doesn't depend on any external variable. We could also have stated that lack of dependency explicitly by writing `V1 => null`.

GNATprove computes dependencies of initial values if you don't supply an `Initializes` aspect. However, if you do provide an `Initializes` aspect for a package, it must be com-

plete: you must list every initialized state of the package, along with all its external dependencies.

Note: For more details on `Initializes`, see the [SPARK User's Guide](#)²³.

4.10 Code Examples / Pitfalls

This section contains some code examples to illustrate potential pitfalls.

4.10.1 Example #1

Package `Communication` defines a hidden local package, `Ring_Buffer`, whose capacity is initialized from an external configuration during elaboration.

Listing 25: `configuration.ads`

```
1 package Configuration is
2
3   External_Variable : Natural := 1;
4
5 end Configuration;
```

Listing 26: `communication.ads`

```
1 with Configuration;
2
3 package Communication with
4   Abstract_State => State,
5   Initializes   => (State => Configuration.External_Variable)
6 is
7   function Get_Capacity return Natural;
8
9 private
10
11   package Ring_Buffer with
12     Initializes => (Capacity => Configuration.External_Variable)
13   is
14     Capacity : constant Natural := Configuration.External_Variable;
15   end Ring_Buffer;
16
17 end Communication;
```

Listing 27: `communication.adb`

```
1 package body Communication with
2   Refined_State => (State => Ring_Buffer.Capacity)
3 is
4
5   function Get_Capacity return Natural is
6   begin
7     return Ring_Buffer.Capacity;
8   end Get_Capacity;
```

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²³ https://docs.adacore.com/live/wave/spark2014/html/spark2014_ug/en/source/package_contracts.html#package-initialization

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```

9
10 end Communication;

```

Code block metadata

```

Project: Courses.Intro_To_Spark.State_Abstraction.Example_01
MD5: 207e999f85a5b39fa2b9aebbc836b479

```

Prover output

```

Phase 1 of 2: generation of Global contracts ...
communication.adb:2:41: error: "Capacity" cannot act as constituent of state "State"
↪
communication.adb:2:41: error: missing Part_Of indicator at communication.ads:14,
↪should specify encapsulator "State"
gnatprove: error during generation of Global contracts

```

This example isn't correct. Capacity is declared in the private part of Communication. Therefore, we should have linked it to State by using the Part_Of aspect in its declaration.

4.10.2 Example #2

Let's add Part_Of to the state of hidden local package Ring_Buffer, but this time we hide variable Capacity inside the private part of Ring_Buffer.

Listing 28: configuration.ads

```

1 package Configuration is
2
3   External_Variable : Natural := 1;
4
5 end Configuration;

```

Listing 29: communication.ads

```

1 with Configuration;
2
3 package Communication with
4   Abstract_State => State
5 is
6 private
7
8   package Ring_Buffer with
9     Abstract_State => (B_State with Part_Of => State),
10    Initializes    => (B_State => Configuration.External_Variable)
11 is
12   function Get_Capacity return Natural;
13 private
14   Capacity : constant Natural := Configuration.External_Variable
15     with Part_Of => B_State;
16 end Ring_Buffer;
17
18 end Communication;

```

Listing 30: communication.adb

```

1 package body Communication with
2   Refined_State => (State => Ring_Buffer.B_State)

```

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```
3 is
4
5   package body Ring_Buffer with
6     Refined_State => (B_State => Capacity)
7   is
8     function Get_Capacity return Natural is (Capacity);
9   end Ring_Buffer;
10
11 end Communication;
```

Code block metadata

Project: Courses.Intro_To_Spark.State_Abstraction.Example_02
MD5: b8d31fcfbd11bf305646efe07baeb91b

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
communication.ads:10:06: info: flow dependencies proved

This program is correct and GNATprove is able to verify it.

4.10.3 Example #3

Package Counting defines two counters: Black_Counter and Red_Counter. It provides separate initialization procedures for each, both called from the main procedure.

Listing 31: counting.ads

```
1 package Counting with
2   Abstract_State => State
3 is
4   procedure Reset_Black_Count;
5   procedure Reset_Red_Count;
6 end Counting;
```

Listing 32: counting.adb

```
1 package body Counting with
2   Refined_State => (State => (Black_Counter, Red_Counter))
3 is
4   Black_Counter, Red_Counter : Natural;
5
6   procedure Reset_Black_Count is
7   begin
8     Black_Counter := 0;
9   end Reset_Black_Count;
10
11  procedure Reset_Red_Count is
12  begin
13    Red_Counter := 0;
14  end Reset_Red_Count;
15 end Counting;
```

Listing 33: main.adb

```

1 with Counting; use Counting;
2
3 procedure Main is
4 begin
5     Reset_Black_Count;
6     Reset_Red_Count;
7 end Main;

```

Code block metadata

```

Project: Courses.Intro_To_Spark.State_Abstraction.Example_03
MD5: bc2d7ccd7419d34f7156a16dfc484229

```

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
main.adb:5:04: medium: "Counting.State" might not be initialized after elaboration,
↳of main program "Main"
counting.ads:2:21: warning: no procedure exists that can initialize abstract state
↳"Counting.State"
gnatprove: unproved check messages considered as errors

```

This program doesn't read any uninitialized data, but GNATprove fails to verify that. This is because we provided a state abstraction for package Counting, so flow analysis computes the effects of subprograms in terms of this state abstraction and thus considers State to be an in-out global consisting of both Black_Counter and Red_Counter. So it issues the message requiring that State be initialized after elaboration as well as the warning that no procedure in package Counting can initialize its state.

4.10.4 Example #4

Let's remove the abstract state on package Counting.

Listing 34: counting.ads

```

1 package Counting is
2     procedure Reset_Black_Count;
3     procedure Reset_Red_Count;
4 end Counting;

```

Listing 35: counting.adb

```

1 package body Counting is
2     Black_Counter, Red_Counter : Natural;
3
4     procedure Reset_Black_Count is
5     begin
6         Black_Counter := 0;
7     end Reset_Black_Count;
8
9     procedure Reset_Red_Count is
10    begin
11        Red_Counter := 0;
12    end Reset_Red_Count;
13 end Counting;

```

Listing 36: main.adb

```
1 with Counting; use Counting;
2
3 procedure Main is
4 begin
5     Reset_Black_Count;
6     Reset_Red_Count;
7 end Main;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.State_Abstraction.Example_04
MD5: 3ddd934b6ede6df7b823e46828694d12
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
```

This example is correct. Because we didn't provide a state abstraction, GNATprove reasons in terms of variables, instead of states, and proves data initialization without any problem.

4.10.5 Example #5

Let's restore the abstract state to package Counting, but this time provide a procedure `Reset_All` that calls the initialization procedures `Reset_Black_Counter` and `Reset_Red_Counter`.

Listing 37: counting.ads

```
1 package Counting with
2   Abstract_State => State
3 is
4   procedure Reset_Black_Count with Global => (In_Out => State);
5   procedure Reset_Red_Count   with Global => (In_Out => State);
6   procedure Reset_All         with Global => (Output => State);
7 end Counting;
```

Listing 38: counting.adb

```
1 package body Counting with
2   Refined_State => (State => (Black_Counter, Red_Counter))
3 is
4   Black_Counter, Red_Counter : Natural;
5
6   procedure Reset_Black_Count is
7   begin
8     Black_Counter := 0;
9   end Reset_Black_Count;
10
11  procedure Reset_Red_Count is
12  begin
13    Red_Counter := 0;
14  end Reset_Red_Count;
15
16  procedure Reset_All is
17  begin
18    Reset_Black_Count;
```

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```

19     Reset_Red_Count;
20     end Reset_All;
21 end Counting;

```

Code block metadata

```

Project: Courses.Intro_To_Spark.State_Abstraction.Example_05
MD5: d123ccc644fe6999699388708f2ecf89

```

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
counting.ads:4:37: info: data dependencies proved
counting.ads:5:37: info: data dependencies proved
counting.ads:6:14: info: initialization of "Black_Counter" constituent of "State"
↳proved
counting.ads:6:14: info: initialization of "Red_Counter" constituent of "State"
↳proved
counting.ads:6:37: info: data dependencies proved

```

This example is correct. Flow analysis computes refined versions of Global contracts for internal calls and uses these to verify that `Reset_All` indeed properly initializes `State`. The `Refined_Global` and `Global` annotations are not mandatory and can be computed by `GNATprove`.

4.10.6 Example #6

Let's consider yet another version of our abstract stack unit.

Listing 39: stack.ads

```

1 package Stack with
2   Abstract_State => The_Stack
3 is
4   pragma Unevaluated_Use_Of_Old (Allow);
5
6   type Element is new Integer;
7
8   type Element_Array is array (Positive range <>) of Element;
9   Max : constant Natural := 100;
10  subtype Length_Type is Natural range 0 .. Max;
11
12  procedure Push (E : Element) with
13    Post =>
14      not Is_Empty and
15      (if Is_Full'Old then The_Stack = The_Stack'Old else Peek = E);
16
17  function Peek return Element with Pre => not Is_Empty;
18  function Is_Full return Boolean;
19  function Is_Empty return Boolean;
20 end Stack;

```

Listing 40: stack.adb

```

1 package body Stack with
2   Refined_State => (The_Stack => (Top, Content))
3 is
4   Top : Length_Type := 0;

```

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```

5   Content : Element_Array (1 .. Max) := (others => 0);
6
7   procedure Push (E : Element) is
8   begin
9       Top           := Top + 1;
10      Content (Top) := E;
11  end Push;
12
13  function Peek      return Element is (Content (Top));
14  function Is_Full   return Boolean is (Top >= Max);
15  function Is_Empty  return Boolean is (Top = 0);
16 end Stack;

```

Code block metadata

Project: Courses.Intro_To_Spark.State_Abstraction.Example_06
MD5: 9da2b74da203a639dc66b2d33cbd500d

Build output

```

stack.ads:15:39: error: there is no applicable operator "=" for package or_
↳procedure name
gprbuild: *** compilation phase failed

```

Prover output

```

Phase 1 of 2: generation of Global contracts ...
stack.ads:15:39: error: there is no applicable operator "=" for package or_
↳procedure name
gnatprove: error during generation of Global contracts

```

This example isn't correct. There's a compilation error in Push's postcondition: The Stack is a state abstraction, not a variable, and therefore can't be used in an expression.

4.10.7 Example #7

In this version of our abstract stack unit, a copy of the stack is returned by function Get_Stack, which we call in the postcondition of Push to specify that the stack shouldn't be modified if it's full. We also assert that after we push an element on the stack, either the stack is unchanged (if it was already full) or its top element is equal to the element just pushed.

Listing 41: stack.ads

```

1  package Stack with
2     Abstract_State => The_Stack
3  is
4     pragma Unevaluated_Use_Of_Old (Allow);
5
6     type Stack_Model is private;
7
8     type Element is new Integer;
9     type Element_Array is array (Positive range <>) of Element;
10    Max : constant Natural := 100;
11    subtype Length_Type is Natural range 0 .. Max;
12
13    function Peek      return Element with Pre => not Is_Empty;
14    function Is_Full   return Boolean;
15    function Is_Empty  return Boolean;

```

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```

16  function Get_Stack return Stack_Model;
17
18  procedure Push (E : Element) with
19    Post => not Is_Empty and
20    (if Is_Full'Old then Get_Stack = Get_Stack'Old else Peek = E);
21
22 private
23
24  type Stack_Model is record
25    Top      : Length_Type := 0;
26    Content : Element_Array (1 .. Max) := (others => 0);
27  end record;
28
29 end Stack;

```

Listing 42: stack.adb

```

1  package body Stack with
2    Refined_State => (The_Stack => (Top, Content))
3  is
4    Top      : Length_Type := 0;
5    Content : Element_Array (1 .. Max) := (others => 0);
6
7    procedure Push (E : Element) is
8    begin
9      if Top >= Max then
10         return;
11       end if;
12       Top      := Top + 1;
13       Content (Top) := E;
14     end Push;
15
16     function Peek      return Element is (Content (Top));
17     function Is_Full   return Boolean is (Top >= Max);
18     function Is_Empty return Boolean is (Top = 0);
19
20     function Get_Stack return Stack_Model is (Stack_Model'(Top, Content));
21
22 end Stack;

```

Listing 43: use_stack.adb

```

1  with Stack; use Stack;
2
3  procedure Use_Stack (E : Element) with
4    Pre => not Is_Empty
5  is
6    F : Element := Peek;
7  begin
8    Push (E);
9    pragma Assert (Peek = E or Peek = F);
10 end Use_Stack;

```

Code block metadata

Project: Courses.Intro_To_Spark.State_Abstraction.Example_07
MD5: 4831aa7f018f2e2d4e6d102095f8f631

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
use_stack.adb:9:19: medium: assertion might fail [possible fix: precondition of
↳subprogram at line 3 should mention E]
gnatprove: unproved check messages considered as errors
```

This program is correct, but GNATprove can't prove the assertion in `Use_Stack`. Indeed, even if `Get_Stack` is an expression function, its body isn't visible outside of `Stack`'s body, where it's defined.

4.10.8 Example #8

Let's move the definition of `Get_Stack` and other expression functions inside the private part of the spec of `Stack`.

Listing 44: `stack.ads`

```
1 package Stack with
2   Abstract_State => The_Stack
3 is
4   pragma Unevaluated_Use_Of_Old (Allow);
5
6   type Stack_Model is private;
7
8   type Element is new Integer;
9   type Element_Array is array (Positive range <>) of Element;
10  Max : constant Natural := 100;
11  subtype Length_Type is Natural range 0 .. Max;
12
13  function Peek      return Element with Pre => not Is_Empty;
14  function Is_Full   return Boolean;
15  function Is_Empty  return Boolean;
16  function Get_Stack return Stack_Model;
17
18  procedure Push (E : Element) with
19    Post => not Is_Empty and
20      (if Is_Full'Old then Get_Stack = Get_Stack'Old else Peek = E);
21
22 private
23
24  Top      : Length_Type := 0 with Part_Of => The_Stack;
25  Content : Element_Array (1 .. Max) := (others => 0) with
26    Part_Of => The_Stack;
27
28  type Stack_Model is record
29    Top      : Length_Type := 0;
30    Content : Element_Array (1 .. Max) := (others => 0);
31  end record;
32
33  function Peek      return Element   is (Content (Top));
34  function Is_Full   return Boolean   is (Top >= Max);
35  function Is_Empty  return Boolean   is (Top = 0);
36
37  function Get_Stack return Stack_Model is (Stack_Model'(Top, Content));
38
39 end Stack;
```

Listing 45: stack.adb

```

1 package body Stack with
2   Refined_State => (The_Stack => (Top, Content))
3 is
4
5   procedure Push (E : Element) is
6   begin
7     if Top >= Max then
8       return;
9     end if;
10    Top           := Top + 1;
11    Content (Top) := E;
12  end Push;
13
14 end Stack;
```

Listing 46: use_stack.adb

```

1 with Stack; use Stack;
2
3 procedure Use_Stack (E : Element) with
4   Pre => not Is_Empty
5 is
6   F : Element := Peek;
7 begin
8   Push (E);
9   pragma Assert (Peek = E or Peek = F);
10 end Use_Stack;
```

Code block metadata

Project: Courses.Intro_To_Spark.State_Abstraction.Example_08
MD5: 7e5204d3f69e71c212e7263906a89da4

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
use_stack.adb:6:19: info: precondition proved
use_stack.adb:9:19: info: precondition proved
use_stack.adb:9:19: info: assertion proved
use_stack.adb:9:31: info: precondition proved
stack.adb:10:30: info: range check proved
stack.adb:11:16: info: index check proved
stack.ads:19:14: info: postcondition proved
stack.ads:20:60: info: precondition proved
stack.ads:33:55: info: index check proved
```

This example is correct. GNATprove can verify the assertion in Use_Stack because it has visibility to Get_Stack's body.

4.10.9 Example #9

Package `Data` defines three variables, `Data_1`, `Data_2` and `Data_3`, that are initialized at elaboration (in `Data`'s package body) from an external interface that reads the file system.

Listing 47: `external_interface.ads`

```
1 package External_Interface with
2   Abstract_State => File_System,
3   Initializes   => File_System
4 is
5   type Data_Type_1 is new Integer;
6   type Data_Type_2 is new Integer;
7   type Data_Type_3 is new Integer;
8
9   type Data_Record is record
10    Field_1 : Data_Type_1;
11    Field_2 : Data_Type_2;
12    Field_3 : Data_Type_3;
13  end record;
14
15  procedure Read_Data (File_Name : String; Data : out Data_Record)
16    with Global => File_System;
17 end External_Interface;
```

Listing 48: `data.ads`

```
1 with External_Interface; use External_Interface;
2
3 package Data with
4   Initializes => (Data_1, Data_2, Data_3)
5 is
6   pragma Elaborate_Body;
7
8   Data_1 : Data_Type_1;
9   Data_2 : Data_Type_2;
10  Data_3 : Data_Type_3;
11
12 end Data;
```

Listing 49: `data.adb`

```
1 with External_Interface;
2 pragma Elaborate_All (External_Interface);
3
4 package body Data is
5 begin
6   declare
7     Data_Read : Data_Record;
8   begin
9     Read_Data ("data_file_name", Data_Read);
10    Data_1 := Data_Read.Field_1;
11    Data_2 := Data_Read.Field_2;
12    Data_3 := Data_Read.Field_3;
13  end;
14 end Data;
```

Code block metadata

Project: `Courses.Intro_To_Spark.State_Abstraction.Example_09`
MD5: `0ca44501f0c991865ea50d2ef663d992`

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
data.adb:9:07: high: "External_Interface.File_System" must be mentioned as an
↳input of the Initializes aspect of "Data" (SPARK RM 7.1.5(11))
gnatprove: unproved check messages considered as errors
```

This example isn't correct. The dependency between Data_1's, Data_2's, and Data_3's initial values and File_System must be listed in Data's Initializes aspect.

4.10.10 Example #10

Let's remove the Initializes contract on package Data.

Listing 50: external_interface.ads

```
1 package External_Interface with
2   Abstract_State => File_System,
3   Initializes   => File_System
4 is
5   type Data_Type_1 is new Integer;
6   type Data_Type_2 is new Integer;
7   type Data_Type_3 is new Integer;
8
9   type Data_Record is record
10    Field_1 : Data_Type_1;
11    Field_2 : Data_Type_2;
12    Field_3 : Data_Type_3;
13  end record;
14
15  procedure Read_Data (File_Name : String; Data : out Data_Record)
16    with Global => File_System;
17 end External_Interface;
```

Listing 51: data.ads

```
1 with External_Interface; use External_Interface;
2
3 package Data is
4   pragma Elaborate_Body;
5
6   Data_1 : Data_Type_1;
7   Data_2 : Data_Type_2;
8   Data_3 : Data_Type_3;
9
10 end Data;
```

Listing 52: data.adb

```
1 with External_Interface;
2 pragma Elaborate_All (External_Interface);
3
4 package body Data is
5 begin
6   declare
7     Data_Read : Data_Record;
8   begin
9     Read_Data ("data_file_name", Data_Read);
10    Data_1 := Data_Read.Field_1;
```

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```
11     Data_2 := Data_Read.Field_2;  
12     Data_3 := Data_Read.Field_3;  
13     end;  
14 end Data;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.State_Abstraction.Example_10  
MD5: 60cba2c920c7b1031d13c82a982ed0e9
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...  
Phase 2 of 2: analysis of data and information flow ...  
data.adb:7:07: info: initialization of "Data_Read" proved  
external_interface.ads:3:03: info: flow dependencies proved
```

This example is correct. Since `Data` has no `Initializes` aspect, GNATprove computes the set of variables initialized during its elaboration as well as their dependencies.

PROOF OF FUNCTIONAL CORRECTNESS

This section is dedicated to the functional correctness of programs. It presents advanced proof features that you may need to use for the specification and verification of your program's complex properties.

5.1 Beyond Program Integrity

When we speak about the *correctness* of a program or subprogram, we mean the extent to which it complies with its specification. Functional correctness is specifically concerned with properties that involve the relations between the subprogram's inputs and outputs, as opposed to other properties such as running time or memory consumption.

For functional correctness, we usually specify stronger properties than those required to just prove program integrity. When we're involved in a certification processes, we should derive these properties from the requirements of the system, but, especially in non-certification contexts, they can also come from more informal sources, such as the program's documentation, comments in its code, or test oracles.

For example, if one of our goals is to ensure that no runtime error is raised when using the result of the function `Find` below, it may be enough to know that the result is either 0 or in the range of `A`. We can express this as a postcondition of `Find`.

Listing 1: show_find.ads

```
1 package Show_Find is
2
3     type Nat_Array is array (Positive range <>) of Natural;
4
5     function Find (A : Nat_Array; E : Natural) return Natural with
6         Post => Find'Result in 0 | A'Range;
7
8 end Show_Find;
```

Listing 2: show_find.adb

```
1 package body Show_Find is
2
3     function Find (A : Nat_Array; E : Natural) return Natural is
4     begin
5         for I in A'Range loop
6             if A (I) = E then
7                 return I;
8             end if;
9         end loop;
10        return 0;
11    end Find;
```

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```
12  
13 end Show_Find;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Find_1  
MD5: d8f4ace6620fd46af170977c29947289
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...  
Phase 2 of 2: flow analysis and proof ...  
show_find.adb:7:20: info: range check proved  
show_find.ads:6:14: info: postcondition proved
```

In this case, it's automatically proved by GNATprove.

However, to be sure that Find performs the task we expect, we may want to verify more complex properties of that function. For example, we want to ensure it returns an index of A where E is stored and returns 0 only if E is nowhere in A. Again, we can express this as a postcondition of Find.

Listing 3: show_find.ads

```
1 package Show_Find is  
2  
3   type Nat_Array is array (Positive range <>) of Natural;  
4  
5   function Find (A : Nat_Array; E : Natural) return Natural with  
6     Post =>  
7     (if (for all I in A'Range => A (I) /= E)  
8       then Find'Result = 0  
9       else Find'Result in A'Range and then A (Find'Result) = E);  
10  
11 end Show_Find;
```

Listing 4: show_find.adb

```
1 package body Show_Find is  
2  
3   function Find (A : Nat_Array; E : Natural) return Natural is  
4     begin  
5       for I in A'Range loop  
6         if A (I) = E then  
7           return I;  
8         end if;  
9       end loop;  
10      return 0;  
11    end Find;  
12  
13 end Show_Find;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Find_2  
MD5: 8c12b9768228a3ea45ca02199f65057b
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...  
Phase 2 of 2: flow analysis and proof ...
```

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```
show_find.ads:9:14: medium: postcondition might fail, cannot prove Find'Result in A
↳ 'range
gnatprove: unproved check messages considered as errors
```

This time, GNATprove can't prove this postcondition automatically, but we'll see later that we can help GNATprove by providing a loop invariant, which is checked by GNATprove and allows it to automatically prove the postcondition for Find.

Writing at least part of your program's specification in the form of contracts has many advantages. You can execute those contracts during testing, which improves the maintainability of the code by detecting discrepancies between the program and its specification in earlier stages of development. If the contracts are precise enough, you can use them as oracles to decide whether a given test passed or failed. In that case, they can allow you to verify the outputs of specific subprograms while running a larger block of code. This may, in certain contexts, replace the need for you to perform unit testing, instead allowing you to run integration tests with assertions enabled. Finally, if the code is in SPARK, you can also use GNATprove to formally prove these contracts.

The advantage of a formal proof is that it verifies all possible execution paths, something which isn't always possible by running test cases. For example, during testing, the postcondition of the subprogram Find shown below is checked dynamically for the set of inputs for which Find is called in that test, but just for that set.

Listing 5: show_find.ads

```
1 package Show_Find is
2
3   type Nat_Array is array (Positive range <>) of Natural;
4
5   function Find (A : Nat_Array; E : Natural) return Natural with
6     Post =>
7     (if (for all I in A'Range => A (I) /= E)
8       then Find'Result = 0
9       else Find'Result in A'Range and then A (Find'Result) = E);
10
11 end Show_Find;
```

Listing 6: show_find.adb

```
1 package body Show_Find is
2
3   function Find (A : Nat_Array; E : Natural) return Natural is
4   begin
5     for I in A'Range loop
6       if A (I) = E then
7         return I;
8       end if;
9     end loop;
10    return 0;
11  end Find;
12
13 end Show_Find;
```

Listing 7: use_find.adb

```
1 with Ada.Text_IO; use Ada.Text_IO;
2 with Show_Find; use Show_Find;
3
4 procedure Use_Find with
5   SPARK_Mode => Off
```

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```
6 is
7   Seq : constant Nat_Array (1 .. 3) := (1, 5, 3);
8   Res : Natural;
9 begin
10  Res := Find (Seq, 3);
11  Put_Line ("Found 3 in index #" & Natural'Image (Res) & " of array");
12 end Use_Find;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Find_3
MD5: 05283ef7808ee5d8254cfa4b883e639d
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_find.ads:9:14: medium: postcondition might fail, cannot prove Find'Result in A
↳ 'range
gnatprove: unproved check messages considered as errors
```

Runtime output

```
Found 3 in index # 3 of array
```

However, if Find is formally verified, that verification checks its postcondition for all possible inputs. During development, you can attempt such verification earlier than testing since it's performed modularly on a per-subprogram basis. For example, in the code shown above, you can formally verify Use_Find even before you write the body for subprogram Find.

5.2 Advanced Contracts

Contracts for functional correctness are usually more complex than contracts for program integrity, so they more often require you to use the new forms of expressions introduced by the Ada 2012 standard. In particular, quantified expressions, which allow you to specify properties that must hold for all or for at least one element of a range, come in handy when specifying properties of arrays.

As contracts become more complex, you may find it useful to introduce new abstractions to improve the readability of your contracts. Expression functions are a good means to this end because you can retain their bodies in your package's specification.

Finally, some properties, especially those better described as invariants over data than as properties of subprograms, may be cumbersome to express as subprogram contracts. Type predicates, which must hold for every object of a given type, are usually a better match for this purpose. Here's an example.

Listing 8: show_sort.ads

```
1 package Show_Sort is
2
3   type Nat_Array is array (Positive range <>) of Natural;
4
5   function Is_Sorted (A : Nat_Array) return Boolean is
6     (for all I in A'Range =>
7       (if I < A'Last then A (I) <= A (I + 1)));
8   -- Returns True if A is sorted in increasing order.
9
```

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```

10  subtype Sorted_Nat_Array is Nat_Array with
11     Dynamic_Predicate => Is_Sorted (Sorted_Nat_Array);
12     -- Elements of type Sorted_Nat_Array are all sorted.
13
14     Good_Array : Sorted_Nat_Array := (1, 2, 4, 8, 42);
15 end Show_Sort;

```

Code block metadata

```

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Sort
MD5: d3b3d26d62074d11b19d9282cc548c1b

```

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_sort.ads:7:32: info: index check proved
show_sort.ads:7:43: info: overflow check proved
show_sort.ads:7:43: info: index check proved
show_sort.ads:14:37: info: range check proved
show_sort.ads:14:37: info: predicate check proved

```

We can use the subtype `Sorted_Nat_Array` as the type of a variable that must remain sorted throughout the program's execution. Specifying that an array is sorted requires a rather complex expression involving quantifiers, so we abstract away this property as an expression function to improve readability. `Is_Sorted`'s body remains in the package's specification and allows users of the package to retain a precise knowledge of its meaning when necessary. (You must use `Nat_Array` as the type of the operand of `Is_Sorted`. If you use `Sorted_Nat_Array`, you'll get infinite recursion at runtime when assertion checks are enabled since that function is called to check all operands of type `Sorted_Nat_Array`.)

5.2.1 Ghost Code

As the properties you need to specify grow more complex, you may have entities that are only needed because they are used in specifications (contracts). You may find it important to ensure that these entities can't affect the behavior of the program or that they're completely removed from production code. This concept, having entities that are only used for specifications, is usually called having *ghost* code and is supported in SPARK by the Ghost aspect.

You can use Ghost aspects to annotate any entity including variables, types, subprograms, and packages. If you mark an entity as Ghost, GNATprove ensures it can't affect the program's behavior. When the program is compiled with assertions enabled, ghost code is executed like normal code so it can execute the contracts using it. You can also instruct the compiler to not generate code for ghost entities.

Consider the procedure `Do_Something` below, which calls a complex function on its input, `X`, and wants to check that the initial and modified values of `X` are related in that complex way.

Listing 9: show_ghost.ads

```

1  package Show_Ghost is
2
3     type T is record
4         A, B, C, D, E : Boolean;
5     end record;
6

```

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```

7   function Formula (X : T) return Boolean is
8     ((X.A and X.B) or (X.C and (X.D or X.E)));
9
10  function Is_Correct (X, Y : T) return Boolean is
11    (Formula (X) = Formula (Y));
12
13  procedure Do_Something (X : in out T);
14
15  end Show_Ghost;

```

Listing 10: show_ghost.adb

```

1  package body Show_Ghost is
2
3    procedure Do_Some_Complex_Stuff (X : in out T) is
4      begin
5        X := T'(X.B, X.A, X.C, X.E, X.D);
6      end Do_Some_Complex_Stuff;
7
8    procedure Do_Something (X : in out T) is
9      X_Init : constant T := X with Ghost;
10     begin
11       Do_Some_Complex_Stuff (X);
12       pragma Assert (Is_Correct (X_Init, X));
13       -- It is OK to use X_Init inside an assertion.
14     end Do_Something;
15
16  end Show_Ghost;

```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Ghost_1
MD5: 0a6caaec950b3b043a53c18bab3cb39b

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_ghost.adb:12:22: info: assertion proved

```

Do_Something stores the initial value of X in a ghost constant, X_Init. We reference it in an assertion to check that the computation performed by the call to Do_Some_Complex_Stuff modified the value of X in the expected manner.

However, X_Init can't be used in normal code, for example to restore the initial value of X.

Listing 11: show_ghost.ads

```

1  package Show_Ghost is
2
3    type T is record
4      A, B, C, D, E : Boolean;
5    end record;
6
7    function Formula (X : T) return Boolean is
8      ((X.A and X.B) or (X.C and (X.D or X.E)));
9
10   function Is_Correct (X, Y : T) return Boolean is
11     (Formula (X) = Formula (Y));
12

```

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```

13  procedure Do_Something (X : in out T);
14
15  end Show_Ghost;

```

Listing 12: show_ghost.adb

```

1  package body Show_Ghost is
2
3  procedure Do_Some_Complex_Stuff (X : in out T) is
4  begin
5      X := T'(X.B, X.A, X.C, X.E, X.D);
6  end Do_Some_Complex_Stuff;
7
8  procedure Do_Something (X : in out T) is
9      X_Init : constant T := X with Ghost;
10 begin
11     Do_Some_Complex_Stuff (X);
12     pragma Assert (Is_Correct (X_Init, X));
13
14     X := X_Init; -- ERROR
15
16 end Do_Something;
17
18 end Show_Ghost;

```

Listing 13: use_ghost.adb

```

1  with Show_Ghost; use Show_Ghost;
2
3  procedure Use_Ghost is
4      X : T := (True, True, False, False, True);
5  begin
6      Do_Something (X);
7  end Use_Ghost;

```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Ghost_2
MD5: 464bb4bc355a648e2b92940ec80b4717

Build output

```

show_ghost.adb:14:12: error: ghost entity cannot appear in this context
gprbuild: *** compilation phase failed

```

Prover output

```

Phase 1 of 2: generation of Global contracts ...
show_ghost.adb:14:12: error: ghost entity cannot appear in this context
gnatprove: error during generation of Global contracts

```

When compiling this example, the compiler flags the use of `X_Init` as illegal, but more complex cases of interference between ghost and normal code may sometimes only be detected when you run `GNATprove`.

5.2.2 Ghost Functions

Functions used only in specifications are a common occurrence when writing contracts for functional correctness. For example, expression functions used to simplify or factor out common patterns in contracts can usually be marked as ghost.

But ghost functions can do more than improve readability. In real-world programs, it's often the case that some information necessary for functional specification isn't accessible in the package's specification because of abstraction.

Making this information available to users of the packages is generally out of the question because that breaks the abstraction. Ghost functions come in handy in that case since they provide a way to give access to that information without making it available to normal client code.

Let's look at the following example.

Listing 14: stacks.ads

```
1 package Stacks is
2
3   pragma Unevaluated_Use_Of_Old (Allow);
4
5   type Stack is private;
6
7   type Element is new Natural;
8   type Element_Array is array (Positive range <>) of Element;
9   Max : constant Natural := 100;
10
11  function Get_Model (S : Stack) return Element_Array with Ghost;
12  -- Returns an array as a model of a stack.
13
14  procedure Push (S : in out Stack; E : Element) with
15    Pre => Get_Model (S)'Length < Max,
16    Post => Get_Model (S) = Get_Model (S)'Old & E;
17
18 private
19
20  subtype Length_Type is Natural range 0 .. Max;
21
22  type Stack is record
23    Top      : Length_Type := 0;
24    Content : Element_Array (1 .. Max) := (others => 0);
25  end record;
26
27 end Stacks;
```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Ghost_Functions
MD5: e287612bd66753f07ac3eecb36c693de

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...

Here, the type `Stack` is private. To specify the expected behavior of the `Push` procedure, we need to go inside this abstraction and access the values of the elements stored in `S`. For this, we introduce a function `Get_Model` that returns an array as a representation of the stack. However, we don't want code that uses the `Stack` package to use `Get_Model` in normal code since this breaks our stack's abstraction.

Here's an example of trying to break that abstraction in the subprogram Peek below.

Listing 15: stacks.ads

```

1 package Stacks is
2
3   pragma Unevaluated_Use_Of_Old (Allow);
4
5   type Stack is private;
6
7   type Element is new Natural;
8   type Element_Array is array (Positive range <>) of Element;
9   Max : constant Natural := 100;
10
11  function Get_Model (S : Stack) return Element_Array with Ghost;
12  -- Returns an array as a model of a stack.
13
14  procedure Push (S : in out Stack; E : Element) with
15    Pre => Get_Model (S)'Length < Max,
16    Post => Get_Model (S) = Get_Model (S)'Old & E;
17
18  function Peek (S : Stack; I : Positive) return Element is
19    (Get_Model (S) (I)); -- ERROR
20
21 private
22
23   subtype Length_Type is Natural range 0 .. Max;
24
25   type Stack is record
26     Top      : Length_Type := 0;
27     Content  : Element_Array (1 .. Max) := (others => 0);
28   end record;
29
30 end Stacks;
```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Ghost_Model
MD5: c00b5d86c9d0b665ccdda7f68f16f07a

Prover output

```

Phase 1 of 2: generation of Global contracts ...
stacks.ads:19:07: error: ghost entity cannot appear in this context
gnatprove: error during generation of Global contracts
```

We see that marking the function as Ghost achieves this goal: it ensures that the subprogram Get_Model is never used in production code.

5.2.3 Global Ghost Variables

Though it happens less frequently, you may have specifications requiring you to store additional information in global variables that isn't needed in normal code. You should mark these global variables as ghost, allowing the compiler to remove them when assertions aren't enabled. You can use these variables for any purpose within the contracts that make up your specifications. A common scenario is writing specifications for subprograms that modify a complex or private global data structure: you can use these variables to provide a model for that structure that's updated by the ghost code as the program modifies the data structure itself.

You can also use ghost variables to store information about previous runs of subprograms

to specify temporal properties. In the following example, we have two procedures, one that accesses a state A and the other that accesses a state B. We use the ghost variable `Last_Accessed_Is_A` to specify that B can't be accessed twice in a row without accessing A in between.

Listing 16: `call_sequence.ads`

```
1 package Call_Sequence is
2
3   type T is new Integer;
4
5   Last_Accessed_Is_A : Boolean := False with Ghost;
6
7   procedure Access_A with
8     Post => Last_Accessed_Is_A;
9
10  procedure Access_B with
11    Pre  => Last_Accessed_Is_A,
12    Post => not Last_Accessed_Is_A;
13    -- B can only be accessed after A
14
15 end Call_Sequence;
```

Listing 17: `call_sequence.adb`

```
1 package body Call_Sequence is
2
3   procedure Access_A is
4     begin
5       -- ...
6       Last_Accessed_Is_A := True;
7     end Access_A;
8
9   procedure Access_B is
10    begin
11      -- ...
12      Last_Accessed_Is_A := False;
13    end Access_B;
14
15 end Call_Sequence;
```

Listing 18: `main.adb`

```
1 with Call_Sequence; use Call_Sequence;
2
3 procedure Main is
4   begin
5     Access_A;
6     Access_B;
7     Access_B; -- ERROR
8   end Main;
```

Code block metadata

Project: `Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Global_Ghost_Vars`
MD5: `f33fa2ad2bd31eb03d4400c78f22eb71`

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...

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```
main.adb:7:04: medium: precondition might fail
gnatprove: unproved check messages considered as errors
```

Runtime output

```
raised ADA.ASSERTIONS.ASSERTION_ERROR : failed precondition from call_sequence.
↳ads:11
```

Let's look at another example. The specification of a subprogram's expected behavior is sometimes best expressed as a sequence of actions it must perform. You can use global ghost variables that store intermediate values of normal variables to write this sort of specification more easily.

For example, we specify the subprogram `Do_Two_Things` below in two steps, using the ghost variable `V_Interm` to store the intermediate value of `V` between those steps. We could also express this using an existential quantification on the variable `V_Interm`, but it would be impractical to iterate over all integers at runtime and this can't always be written in SPARK because quantification is restricted to `for ... loop` patterns.

Finally, supplying the value of the variable may help the prover verify the contracts.

Listing 19: `action_sequence.ads`

```
1 package Action_Sequence is
2
3   type T is new Integer;
4
5   V_Interm : T with Ghost;
6
7   function First_Thing_Done (X, Y : T) return Boolean with Ghost;
8   function Second_Thing_Done (X, Y : T) return Boolean with Ghost;
9
10  procedure Do_Two_Things (V : in out T) with
11     Post => First_Thing_Done (V'Old, V_Interm)
12     and then Second_Thing_Done (V_Interm, V);
13
14 end Action_Sequence;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Intermediate_Values
MD5: 2ffbd2cb187c0a81423c78e0989d62f0
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
```

Note: For more details on ghost code, see the [SPARK User's Guide](#)²⁴.

²⁴ https://docs.adacore.com/live/wave/spark2014/html/spark2014_ug/en/source/specification_features.html#ghost-code

5.3 Guide Proof

Since properties of interest for functional correctness are more complex than those involved in proofs of program integrity, we expect GNATprove to initially be unable to verify them even though they're valid. You'll find the techniques we discussed in *Debugging Failed Proof Attempts* (page 56) to come in handy here. We now go beyond those techniques and focus on more ways of improving results in the cases where the property is valid but GNATprove can't prove it in a reasonable amount of time.

In those cases, you may want to try and guide GNATprove to either complete the proof or strip it down to a small number of easily-reviewable assumptions. For this purpose, you can add assertions to break complex proofs into smaller steps.

```
pragma Assert (Assertion_Checked_By_The_Tool);
-- info: assertion proved

pragma Assert (Assumption_Validated_By_Other_Means);
-- medium: assertion might fail

pragma Assume (Assumption_Validated_By_Other_Means);
-- The tool does not attempt to check this expression.
-- It is recorded as an assumption.
```

One such intermediate step you may find useful is to try to prove a theoretically-equivalent version of the desired property, but one where you've simplified things for the prover, such as by splitting up different cases or inlining the definitions of functions.

Some intermediate assertions may not be proved by GNATprove either because it's missing some information or because the amount of information available is confusing. You can verify these remaining assertions by other means such as testing (since they're executable) or by review. You can then choose to instruct GNATprove to ignore them, either by turning them into assumptions, as in our example, or by using a `pragma Annotate`. In both cases, the compiler generates code to check these assumptions at runtime when you enable assertions.

5.3.1 Local Ghost Variables

You can use ghost code to enhance what you can express inside intermediate assertions in the same way we did above to enhance our contracts in specifications. In particular, you'll commonly have local variables or constants whose only purpose is to be used in assertions. You'll mostly use these ghost variables to store previous values of variables or expressions you want to refer to in assertions. They're especially useful to refer to initial values of parameters and expressions since the `'Old` attribute is only allowed in postconditions.

In the example below, we want to help GNATprove verify the postcondition of `P`. We do this by introducing a local ghost constant, `X_Init`, to represent this value and writing an assertion in both branches of an `if` statement that repeats the postcondition, but using `X_Init`.

Listing 20: show_local_ghost.ads

```
1 package Show_Local_Ghost is
2
3   type T is new Natural;
4
5   function F (X, Y : T) return Boolean is (X > Y) with Ghost;
6
7   function Condition (X : T) return Boolean is (X mod 2 = 0);
8
```

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```

9   procedure P (X : in out T) with
10      Pre => X < 1_000_000,
11      Post => F (X, X'Old);
12
13 end Show_Local_Ghost;

```

Listing 21: show_local_ghost.adb

```

1  package body Show_Local_Ghost is
2
3      procedure P (X : in out T) is
4          X_Init : constant T := X with Ghost;
5      begin
6          if Condition (X) then
7              X := X + 1;
8              pragma Assert (F (X, X_Init));
9          else
10             X := X * 2;
11             pragma Assert (F (X, X_Init));
12         end if;
13     end P;
14
15 end Show_Local_Ghost;

```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Local_Ghost
MD5: 071ee53a06a6b5880eeee6e9ea06dbcf3

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_local_ghost.adb:7:17: info: overflow check proved
show_local_ghost.adb:8:25: info: assertion proved
show_local_ghost.adb:10:17: info: overflow check proved
show_local_ghost.adb:11:25: info: assertion proved
show_local_ghost.ads:7:52: info: division check proved
show_local_ghost.ads:11:14: info: postcondition proved

```

You can also use local ghost variables for more complex purposes such as building a data structure that serves as witness for a complex property of a subprogram. In our example, we want to prove that the Sort procedure doesn't create new elements, that is, that all the elements present in A after the sort were in A before the sort. This property isn't enough to ensure that a call to Sort produces a value for A that's a permutation of its value before the call (or that the values are indeed sorted). However, it's already complex for a prover to verify because it involves a nesting of quantifiers. To help GNATprove, you may find it useful to store, for each index I, an index J that has the expected property.

```

procedure Sort (A : in out Nat_Array) with
  Post => (for all I in A'Range =>
           (for some J in A'Range => A (I) = A'Old (J)))
is
  Permutation : Index_Array := (1 => 1, 2 => 2, ...) with Ghost;
begin
  ...
end Sort;

```


5.3.2 Ghost Procedures

Ghost procedures can't affect the value of normal variables, so they're mostly used to perform operations on ghost variables or to group together a set of intermediate assertions.

Abstracting away the treatment of assertions and ghost variables inside a ghost procedure has several advantages. First, you're allowed to use these variables in any way you choose in code inside ghost procedures. This isn't the case outside ghost procedures, where the only ghost statements allowed are assignments to ghost variables and calls to ghost procedures.

As an example, the `for` loop contained in `Increase_A` couldn't appear by itself in normal code.

Listing 22: `show_ghost_proc.ads`

```
1 package Show_Ghost_Proc is
2
3     type Nat_Array is array (Integer range <>) of Natural;
4
5     A : Nat_Array (1 .. 100) with Ghost;
6
7     procedure Increase_A with
8         Ghost,
9         Pre => (for all I in A'Range => A (I) < Natural'Last);
10
11 end Show_Ghost_Proc;
```

Listing 23: `show_ghost_proc.adb`

```
1 package body Show_Ghost_Proc is
2
3     procedure Increase_A is
4     begin
5         for I in A'Range loop
6             A (I) := A (I) + 1;
7         end loop;
8     end Increase_A;
9
10 end Show_Ghost_Proc;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Ghost_Proc
MD5: 4b9cfe25011169a0cd3b4a3b03135dc4
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_ghost_proc.adb:6:25: info: overflow check proved
```

Using the abstraction also improves readability by hiding complex code that isn't part of the functional behavior of the subprogram. Finally, it can help GNATprove by abstracting away assertions that would otherwise make its job more complex.

In the example below, calling `Prove_P` with `X` as an operand only adds `P (X)` to the proof context instead of the larger set of assertions required to verify it. In addition, the proof of `P` need only be done once and may be made easier not having any unnecessary information present in its context while verifying it. Also, if GNATprove can't fully verify `Prove_P`, you can review the remaining assumptions more easily since they're in a smaller context.

```

procedure Prove_P (X : T) with Ghost,
  Global => null,
  Post   => P (X);

```

5.3.3 Handling of Loops

When the program involves a loop, you're almost always required to provide additional annotations to allow GNATprove to complete a proof because the verification techniques used by GNATprove don't handle cycles in a subprogram's control flow. Instead, loops are flattened by dividing them into several acyclic parts.

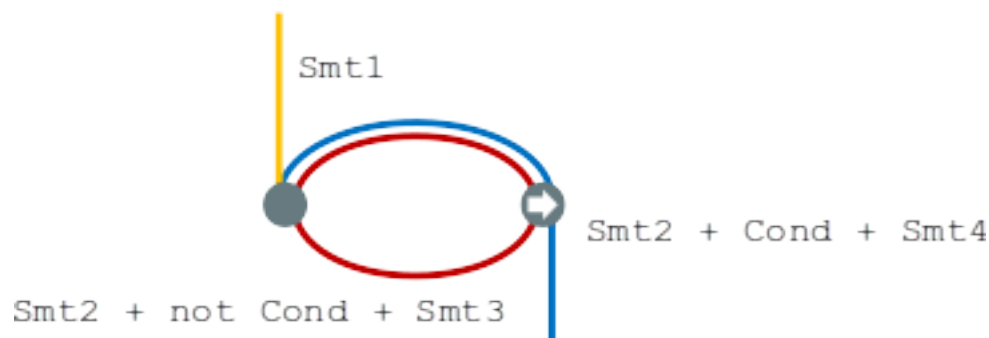
As an example, let's look at a simple loop with an exit condition.

```

Smt1;
loop
  Smt2;
  exit when Cond;
  Smt3;
end loop;
Smt4;

```

As shown below, the control flow is divided into three parts.



The first, shown in yellow, starts earlier in the subprogram and enters the loop statement. The loop itself is divided into two parts. Red represents a complete execution of the loop's body: an execution where the exit condition isn't satisfied. Blue represents the last execution of the loop, which includes some of the subprogram following it. For that path, the exit condition is assumed to hold. The red and blue parts are always executed after the yellow one.

GNATprove analyzes these parts independently since it doesn't have a way to track how variables may have been updated by an iteration of the loop. It forgets everything it knows about those variables from one part when entering another part. However, values of constants and variables that aren't modified in the loop are not an issue.

In other words, handling loops in that way makes GNATprove imprecise when verifying a subprogram involving a loop: it can't verify a property that relies on values of variables modified inside the loop. It won't forget any information it had on the value of constants or unmodified variables, but it nevertheless won't be able to deduce new information about them from the loop.

For example, consider the function Find which iterates over the array A and searches for an element where E is stored in A.

Listing 24: show_find.ads

```

1 package Show_Find is
2

```

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```

3   type Nat_Array is array (Positive range <>) of Natural;
4
5   function Find (A : Nat_Array; E : Natural) return Natural;
6
7 end Show_Find;

```

Listing 25: show_find.adb

```

1 package body Show_Find is
2
3   function Find (A : Nat_Array; E : Natural) return Natural is
4   begin
5     for I in A'Range loop
6       pragma Assert (for all J in A'First .. I - 1 => A (J) /= E);
7       -- assertion is not proved
8       if A (I) = E then
9         return I;
10      end if;
11      pragma Assert (A (I) /= E);
12      -- assertion is proved
13    end loop;
14    return 0;
15  end Find;
16
17 end Show_Find;

```

Code block metadata

```

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Loop
MD5: cb9cd0cb102c3baba3b21a788b6e4ae3

```

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_find.adb:6:51: info: overflow check proved
show_find.adb:6:58: medium: assertion might fail, cannot prove A (J) /= E
↳[possible fix: subprogram at show_find.ads:5 should mention A and E in a
↳precondition]
show_find.adb:6:61: info: index check proved
show_find.adb:9:20: info: range check proved
show_find.adb:11:25: info: assertion proved
gnatprove: unproved check messages considered as errors

```

At the end of each loop iteration, GNATprove knows that the value stored at index I in A must not be E . (If it were, the loop wouldn't have reached the end of the iteration.) This proves the second assertion. But it's unable to aggregate this information over multiple loop iterations to deduce that it's true for all the indexes smaller than I , so it can't prove the first assertion.

5.3.4 Loop Invariants

To overcome these limitations, you can provide additional information to GNATprove in the form of a *loop invariant*. In SPARK, a loop invariant is a Boolean expression which holds true at every iteration of the loop. Like other assertions, you can have it checked at runtime by compiling the program with assertions enabled.

The major difference between loop invariants and other assertions is the way it's treated for proofs. GNATprove performs the proof of a loop invariant in two steps: first, it checks that it holds for the first iteration of the loop and then it checks that it holds in an arbitrary iteration assuming it held in the previous iteration. This is called *proof by induction*²⁵.

As an example, let's add a loop invariant to the Find function stating that the first element of A is not E.

Listing 26: show_find.ads

```

1 package Show_Find is
2
3     type Nat_Array is array (Positive range <>) of Natural;
4
5     function Find (A : Nat_Array; E : Natural) return Natural;
6
7 end Show_Find;
```

Listing 27: show_find.adb

```

1 package body Show_Find is
2
3     function Find (A : Nat_Array; E : Natural) return Natural is
4     begin
5         for I in A'Range loop
6             pragma Loop_Invariant (A (A'First) /= E);
7             -- loop invariant not proved in first iteration
8             -- but preservation of loop invariant is proved
9             if A (I) = E then
10                return I;
11            end if;
12        end loop;
13        return 0;
14    end Find;
15
16 end Show_Find;
```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Loop_Invariant_1
MD5: 8d5fefdc9deacd4eb50850be91fbefe

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_find.adb:6:33: info: loop invariant preservation proved
show_find.adb:6:33: medium: loop invariant might fail in first iteration [possible_
↳fix: subprogram at show_find.ads:5 should mention A and E in a precondition]
show_find.adb:6:37: info: index check proved
show_find.adb:10:20: info: range check proved
gnatprove: unproved check messages considered as errors
```

²⁵ https://en.wikipedia.org/wiki/Mathematical_induction

To verify this invariant, GNATprove generates two checks. The first checks that the assertion holds in the first iteration of the loop. This isn't verified by GNATprove. And indeed there's no reason to expect the first element of A to always be different from E in this iteration. However, the second check is proved: it's easy to deduce that if the first element of A was not E in a given iteration it's still not E in the next. However, if we move the invariant to the end of the loop, then it is successfully verified by GNATprove.

Not only do loop invariants allow you to verify complex properties of loops, but GNATprove also uses them to verify other properties, such as the absence of runtime errors over both the loop's body and the statements following the loop. More precisely, when verifying a runtime check or other assertion there, GNATprove assumes that the last occurrence of the loop invariant preceding the check or assertion is true.

Let's look at a version of Find where we use a loop invariant instead of an assertion to state that none of the array elements seen so far are equal to E.

Listing 28: show_find.ads

```
1 package Show_Find is
2
3   type Nat_Array is array (Positive range <>) of Natural;
4
5   function Find (A : Nat_Array; E : Natural) return Natural;
6
7 end Show_Find;
```

Listing 29: show_find.adb

```
1 package body Show_Find is
2
3   function Find (A : Nat_Array; E : Natural) return Natural is
4   begin
5     for I in A'Range loop
6       pragma Loop_Invariant
7         (for all J in A'First .. I - 1 => A (J) /= E);
8       if A (I) = E then
9         return I;
10      end if;
11    end loop;
12    pragma Assert (for all I in A'Range => A (I) /= E);
13    return 0;
14  end Find;
15
16 end Show_Find;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Loop_Invariant_2
MD5: 21588161eaddb82f54c3cb3dcc14a6ac
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_find.adb:7:13: info: loop invariant initialization proved
show_find.adb:7:13: info: loop invariant preservation proved
show_find.adb:7:39: info: overflow check proved
show_find.adb:7:49: info: index check proved
show_find.adb:9:20: info: range check proved
show_find.adb:12:22: info: assertion proved
show_find.adb:12:49: info: index check proved
```

This version is fully verified by GNATprove! This time, it proves that the loop invariant holds

in every iteration of the loop (separately proving this property for the first iteration and then for the following iterations). It also proves that none of the elements of A are equal to E after the loop exits by assuming that the loop invariant holds in the last iteration of the loop.

Note: For more details on loop invariants, see the [SPARK User's Guide](#)²⁶.

Finding a good loop invariant can turn out to be quite a challenge. To make this task easier, let's review the four good properties of a good loop invariant:

Property	Description
INIT	It should be provable in the first iteration of the loop.
INSIDE	It should allow proving the absence of run-time errors and local assertions inside the loop.
AFTER	It should allow proving absence of run-time errors, local assertions, and the subprogram postcondition after the loop.
PRE-SERVE	It should be provable after the first iteration of the loop.

Let's look at each of these in turn. First, the loop invariant should be provable in the first iteration of the loop (INIT). If your invariant fails to achieve this property, you can debug the loop invariant's initialization like any failing proof attempt using strategies for [Debugging Failed Proof Attempts](#) (page 56).

Second, the loop invariant should be precise enough to allow GNATprove to prove absence of runtime errors in both statements from the loop's body (INSIDE) and those following the loop (AFTER). To do this, you should remember that all information concerning a variable modified in the loop that's not included in the invariant is forgotten by GNATprove. In particular, you should take care to include in your invariant what's usually called the loop's *frame condition*, which lists properties of variables that are true throughout the execution of the loop even though those variables are modified by the loop.

Finally, the loop invariant should be precise enough to prove that it's preserved through successive iterations of the loop (PRESERVE). This is generally the trickiest part. To understand why GNATprove hasn't been able to verify the preservation of a loop invariant you provided, you may find it useful to repeat it as local assertions throughout the loop's body to determine at which point it can no longer be proved.

As an example, let's look at a loop that iterates through an array A and applies a function F to each of its elements.

Listing 30: show_map.ads

```

1 package Show_Map is
2
3   type Nat_Array is array (Positive range <>) of Natural;
4
5   function F (V : Natural) return Natural is
6     (if V /= Natural'Last then V + 1 else V);
7
8   procedure Map (A : in out Nat_Array);
9
10 end Show_Map;
```

²⁶ https://docs.adacore.com/live/wave/spark2014/html/spark2014_ug/en/source/assertion_pragmas.html#loop-invariants

Listing 31: show_map.adb

```

1 package body Show_Map is
2
3   procedure Map (A : in out Nat_Array) is
4     A_I : constant Nat_Array := A with Ghost;
5   begin
6     for K in A'Range loop
7       A (K) := F (A (K));
8       pragma Loop_Invariant
9         (for all J in A'First .. K => A (J) = F (A'Loop_Entry (J)));
10    end loop;
11    pragma Assert (for all K in A'Range => A (K) = F (A_I (K)));
12  end Map;
13
14 end Show_Map;

```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Loop_Invariant_3
MD5: 1a4583c9b2b772f79bcf29cff0caa96a

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_map.adb:9:13: info: loop invariant initialization proved
show_map.adb:9:13: info: loop invariant preservation proved
show_map.adb:9:45: info: index check proved
show_map.adb:9:67: info: index check proved
show_map.adb:11:22: info: assertion proved
show_map.adb:11:49: info: index check proved
show_map.adb:11:62: info: index check proved
show_map.ads:6:35: info: overflow check proved

```

After the loop, each element of *A* should be the result of applying *F* to its previous value. We want to prove this. To specify this property, we copy the value of *A* before the loop into a ghost variable, *A_I*. Our loop invariant states that the element at each index less than *K* has been modified in the expected way. We use the *Loop_Entry* attribute to refer to the value of *A* on entry of the loop instead of using *A_I*.

Does our loop invariant have the four properties of a good loop-invariant? When launching GNATprove, we see that INIT is fulfilled: the invariant's initialization is proved. So are INSIDE and AFTER: no potential runtime errors are reported and the assertion following the loop is successfully verified.

The situation is slightly more complex for the PRESERVE property. GNATprove manages to prove that the invariant holds after the first iteration thanks to the automatic generation of frame conditions. It was able to do this because it completes the provided loop invariant with the following frame condition stating what part of the array hasn't been modified so far:

```

pragma Loop_Invariant
  (for all J in K .. A'Last => A (J) = (if J > K then A'Loop_Entry (J)));

```

GNATprove then uses both our and the internally-generated loop invariants to prove PRESERVE. However, in more complex cases, the heuristics used by GNATprove to generate the frame condition may not be sufficient and you'll have to provide one as a loop invariant. For example, consider a version of *Map* where the result of applying *F* to an element at index *K* is stored at index *K-1*:

Listing 32: show_map.ads

```

1 package Show_Map is
2
3   type Nat_Array is array (Positive range <>) of Natural;
4
5   function F (V : Natural) return Natural is
6     (if V /= Natural'Last then V + 1 else V);
7
8   procedure Map (A : in out Nat_Array);
9
10 end Show_Map;

```

Listing 33: show_map.adb

```

1 package body Show_Map is
2
3   procedure Map (A : in out Nat_Array) is
4     A_I : constant Nat_Array := A with Ghost;
5     begin
6       for K in A'Range loop
7         if K /= A'First then
8           A (K - 1) := F (A (K));
9         end if;
10        pragma Loop_Invariant
11          (for all J in A'First .. K =>
12            (if J /= A'First then A (J - 1) = F (A'Loop_Entry (J))));
13        -- pragma Loop_Invariant
14        -- (for all J in K .. A'Last => A (J) = A'Loop_Entry (J));
15      end loop;
16      pragma Assert (for all K in A'Range =>
17        (if K /= A'First then A (K - 1) = F (A_I (K))));
18    end Map;
19
20 end Show_Map;

```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Loop_Invariant_4
MD5: 6c51768547d3baa2c19d0e33959388fe

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_map.adb:8:18: info: overflow check proved
show_map.adb:8:18: info: index check proved
show_map.adb:11:13: info: loop invariant initialization proved
show_map.adb:12:36: medium: loop invariant might not be preserved by an arbitrary_
↪ iteration, cannot prove A (J - 1) = F (A'Loop_Entry (J))
show_map.adb:12:41: info: overflow check proved
show_map.adb:12:41: info: index check proved
show_map.adb:12:65: info: index check proved
show_map.adb:16:22: info: assertion proved
show_map.adb:17:50: info: overflow check proved
show_map.adb:17:50: info: index check proved
show_map.adb:17:65: info: index check proved
show_map.ads:6:35: info: overflow check proved
gnatprove: unproved check messages considered as errors

```

You need to uncomment the second loop invariant containing the frame condition in order to prove the assertion after the loop.

Note: For more details on how to write a loop invariant, see the [SPARK User's Guide](#)²⁷.

5.4 Code Examples / Pitfalls

This section contains some code examples and pitfalls.

5.4.1 Example #1

We implement a ring buffer inside an array `Content`, where the contents of a ring buffer of length `Length` are obtained by starting at index `First` and possibly wrapping around the end of the buffer. We use a ghost function `Get_Model` to return the contents of the ring buffer for use in contracts.

Listing 34: ring_buffer.ads

```
1 package Ring_Buffer is
2
3   Max_Size : constant := 100;
4
5   type Nat_Array is array (Positive range <>) of Natural;
6
7   function Get_Model return Nat_Array with Ghost;
8
9   procedure Push_Last (E : Natural) with
10     Pre  => Get_Model'Length < Max_Size,
11     Post => Get_Model'Length = Get_Model'Old'Length + 1;
12
13 end Ring_Buffer;
```

Listing 35: ring_buffer.adb

```
1 package body Ring_Buffer is
2
3   subtype Length_Range is Natural range 0 .. Max_Size;
4   subtype Index_Range  is Natural range 1 .. Max_Size;
5
6   Content : Nat_Array (1 .. Max_Size) := (others => 0);
7   First   : Index_Range := 1;
8   Length  : Length_Range := 0;
9
10  function Get_Model return Nat_Array with
11    Refined_Post => Get_Model'Result'Length = Length
12  is
13    Size   : constant Length_Range := Length;
14    Result : Nat_Array (1 .. Size) := (others => 0);
15  begin
16    if First + Length - 1 <= Max_Size then
17      Result := Content (First .. First + Length - 1);
18    else
19      declare
20        Len : constant Length_Range := Max_Size - First + 1;
21      begin
22        Result (1 .. Len) := Content (First .. Max_Size);
```

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²⁷ https://docs.adacore.com/live/wave/spark2014/html/spark2014_ug/en/source/how_to_write_loop_invariants.html

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```

23         Result (Len + 1 .. Length) := Content (1 .. Length - Len);
24     end;
25 end if;
26 return Result;
27 end Get_Model;
28
29 procedure Push_Last (E : Natural) is
30 begin
31     if First + Length <= Max_Size then
32         Content (First + Length) := E;
33     else
34         Content (Length - Max_Size + First) := E;
35     end if;
36     Length := Length + 1;
37 end Push_Last;
38
39 end Ring_Buffer;

```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Example_01
MD5: 3afd7d58f97001618acc05062115f1a3

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
ring_buffer.adb:11:22: info: refined post proved
ring_buffer.adb:11:38: info: range check proved
ring_buffer.adb:14:07: info: range check proved
ring_buffer.adb:14:41: info: length check proved
ring_buffer.adb:17:17: info: length check proved
ring_buffer.adb:17:20: info: range check proved
ring_buffer.adb:17:20: info: length check proved
ring_buffer.adb:20:61: info: range check proved
ring_buffer.adb:22:13: info: range check proved
ring_buffer.adb:22:31: info: length check proved
ring_buffer.adb:22:34: info: range check proved
ring_buffer.adb:22:34: info: length check proved
ring_buffer.adb:23:13: info: range check proved
ring_buffer.adb:23:40: info: length check proved
ring_buffer.adb:23:43: info: range check proved
ring_buffer.adb:23:43: info: length check proved
ring_buffer.adb:32:25: info: index check proved
ring_buffer.adb:34:37: info: index check proved
ring_buffer.adb:36:24: info: range check proved
ring_buffer.ads:11:14: info: postcondition proved

```

This is correct: `Get_Model` is used only in contracts. Calls to `Get_Model` make copies of the buffer's contents, which isn't efficient, but is fine because `Get_Model` is only used for verification, not in production code. We enforce this by making it a ghost function. We'll produce the final production code with appropriate compiler switches (i.e., not using `-gnata`) that ensure assertions are ignored.

5.4.2 Example #2

Instead of using a ghost function, `Get_Model`, to retrieve the contents of the ring buffer, we're now using a global ghost variable, `Model`.

Listing 36: ring_buffer.ads

```
1 package Ring_Buffer is
2
3   Max_Size : constant := 100;
4   subtype Length_Range is Natural range 0 .. Max_Size;
5   subtype Index_Range is Natural range 1 .. Max_Size;
6
7   type Nat_Array is array (Positive range <>) of Natural;
8
9   type Model_Type (Length : Length_Range := 0) is record
10     Content : Nat_Array (1 .. Length);
11   end record
12     with Ghost;
13
14   Model : Model_Type with Ghost;
15
16   function Valid_Model return Boolean;
17
18   procedure Push_Last (E : Natural) with
19     Pre => Valid_Model
20     and then Model.Length < Max_Size,
21     Post => Model.Length = Model.Length'Old + 1;
22
23 end Ring_Buffer;
```

Listing 37: ring_buffer.adb

```
1 package body Ring_Buffer is
2
3   Content : Nat_Array (1 .. Max_Size) := (others => 0);
4   First : Index_Range := 1;
5   Length : Length_Range := 0;
6
7   function Valid_Model return Boolean is
8     (Model.Content.Length = Length);
9
10  procedure Push_Last (E : Natural) is
11  begin
12    if First + Length <= Max_Size then
13      Content (First + Length) := E;
14    else
15      Content (Length - Max_Size + First) := E;
16    end if;
17    Length := Length + 1;
18  end Push_Last;
19
20 end Ring_Buffer;
```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Example_02
MD5: 144f58bd95cd460e4ed388d4f3351fe3

Build output

```
ring_buffer.adb:8:08: error: ghost entity cannot appear in this context
gprbuild: *** compilation phase failed
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
ring_buffer.adb:8:08: error: ghost entity cannot appear in this context
gnatprove: error during generation of Global contracts
```

This example isn't correct. `Model`, which is a ghost variable, must not influence the return value of the normal function `Valid_Model`. Since `Valid_Model` is only used in specifications, we should have marked it as `Ghost`. Another problem is that `Model` needs to be updated inside `Push_Last` to reflect the changes to the ring buffer.

5.4.3 Example #3

Let's mark `Valid_Model` as `Ghost` and update `Model` inside `Push_Last`.

Listing 38: ring_buffer.ads

```
1 package Ring_Buffer is
2
3   Max_Size : constant := 100;
4   subtype Length_Range is Natural range 0 .. Max_Size;
5   subtype Index_Range is Natural range 1 .. Max_Size;
6
7   type Nat_Array is array (Positive range <>) of Natural;
8
9   type Model_Type (Length : Length_Range := 0) is record
10     Content : Nat_Array (1 .. Length);
11   end record
12     with Ghost;
13
14   Model : Model_Type with Ghost;
15
16   function Valid_Model return Boolean with Ghost;
17
18   procedure Push_Last (E : Natural) with
19     Pre => Valid_Model
20     and then Model.Length < Max_Size,
21     Post => Model.Length = Model.Length'Old + 1;
22
23 end Ring_Buffer;
```

Listing 39: ring_buffer.adb

```
1 package body Ring_Buffer is
2
3   Content : Nat_Array (1 .. Max_Size) := (others => 0);
4   First   : Index_Range := 1;
5   Length  : Length_Range := 0;
6
7   function Valid_Model return Boolean is
8     (Model.Content.Length = Length);
9
10  procedure Push_Last (E : Natural) is
11  begin
12    if First + Length <= Max_Size then
13      Content (First + Length) := E;
```

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```
14     else
15         Content (Length - Max_Size + First) := E;
16     end if;
17     Length := Length + 1;
18     Model := (Length => Model.Length + 1,
19              Content => Model.Content & E);
20 end Push_Last;
21
22 end Ring_Buffer;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Example_03
MD5: 08b74f5fe560d238550a06c6323959cf
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
ring_buffer.adb:8:21: info: range check proved
ring_buffer.adb:13:25: info: index check proved
ring_buffer.adb:15:37: info: index check proved
ring_buffer.adb:17:24: info: range check proved
ring_buffer.adb:18:13: info: discriminant check proved
ring_buffer.adb:18:41: info: range check proved
ring_buffer.adb:19:42: info: range check proved
ring_buffer.adb:19:42: info: length check proved
ring_buffer.ads:10:07: info: range check proved
ring_buffer.ads:21:14: info: postcondition proved
```

This example is correct. The ghost variable `Model` can be referenced both from the body of the ghost function `Valid_Model` and the non-ghost procedure `Push_Last` as long as it's only used in ghost statements.

5.4.4 Example #4

We're now modifying `Push_Last` to share the computation of the new length between the operational and ghost code.

Listing 40: ring_buffer.ads

```
1 package Ring_Buffer is
2
3     Max_Size : constant := 100;
4     subtype Length_Range is Natural range 0 .. Max_Size;
5     subtype Index_Range is Natural range 1 .. Max_Size;
6
7     type Nat_Array is array (Positive range <>) of Natural;
8
9     type Model_Type (Length : Length_Range := 0) is record
10         Content : Nat_Array (1 .. Length);
11     end record
12     with Ghost;
13
14     Model : Model_Type with Ghost;
15
16     function Valid_Model return Boolean with Ghost;
17
18     procedure Push_Last (E : Natural) with
```

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```

19     Pre => Valid_Model
20     and then Model.Length < Max_Size,
21     Post => Model.Length = Model.Length'Old + 1;
22
23 end Ring_Buffer;

```

Listing 41: ring_buffer.adb

```

1 package body Ring_Buffer is
2
3     Content : Nat_Array (1 .. Max_Size) := (others => 0);
4     First   : Index_Range := 1;
5     Length  : Length_Range := 0;
6
7     function Valid_Model return Boolean is
8         (Model.Content'Length = Length);
9
10    procedure Push_Last (E : Natural) is
11        New_Length : constant Length_Range := Model.Length + 1;
12    begin
13        if First + Length <= Max_Size then
14            Content (First + Length) := E;
15        else
16            Content (Length - Max_Size + First) := E;
17        end if;
18        Length := New_Length;
19        Model := (Length => New_Length,
20                Content => Model.Content & E);
21    end Push_Last;
22
23 end Ring_Buffer;

```

Code block metadata

```

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Example_04
MD5: e27f0b4729be72d83f2cb981b1d00412

```

Build output

```

ring_buffer.adb:11:45: error: ghost entity cannot appear in this context
gprbuild: *** compilation phase failed

```

Prover output

```

Phase 1 of 2: generation of Global contracts ...
ring_buffer.adb:11:45: error: ghost entity cannot appear in this context
gnatprove: error during generation of Global contracts

```

This example isn't correct. We didn't mark local constant `New_Length` as `Ghost`, so it can't be computed from the value of ghost variable `Model`. If we made `New_Length` a ghost constant, the compiler would report the problem on the assignment from `New_Length` to `Length`. The correct solution here is to compute `New_Length` from the value of the non-ghost variable `Length`.

5.4.5 Example #5

Let's move the code updating `Model` inside a local ghost procedure, `Update_Model`, but still using a local variable, `New_Length`, to compute the length.

Listing 42: ring_buffer.ads

```

1 package Ring_Buffer is
2
3   Max_Size : constant := 100;
4   subtype Length_Range is Natural range 0 .. Max_Size;
5   subtype Index_Range is Natural range 1 .. Max_Size;
6
7   type Nat_Array is array (Positive range <>) of Natural;
8
9   type Model_Type (Length : Length_Range := 0) is record
10     Content : Nat_Array (1 .. Length);
11   end record
12     with Ghost;
13
14   Model : Model_Type with Ghost;
15
16   function Valid_Model return Boolean with Ghost;
17
18   procedure Push_Last (E : Natural) with
19     Pre  => Valid_Model
20     and then Model.Length < Max_Size,
21     Post => Model.Length = Model.Length'Old + 1;
22
23 end Ring_Buffer;
```

Listing 43: ring_buffer.adb

```

1 package body Ring_Buffer is
2
3   Content : Nat_Array (1 .. Max_Size) := (others => 0);
4   First   : Index_Range := 1;
5   Length  : Length_Range := 0;
6
7   function Valid_Model return Boolean is
8     (Model.Content.Length = Length);
9
10  procedure Push_Last (E : Natural) is
11
12    procedure Update_Model with Ghost is
13      New_Length : constant Length_Range := Model.Length + 1;
14    begin
15      Model := (Length => New_Length,
16              Content => Model.Content & E);
17    end Update_Model;
18
19  begin
20    if First + Length <= Max_Size then
21      Content (First + Length) := E;
22    else
23      Content (Length - Max_Size + First) := E;
24    end if;
25    Length := Length + 1;
26    Update_Model;
27  end Push_Last;
28
29 end Ring_Buffer;
```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Example_05
 MD5: cc97fb35205c9a6de06001cf489f34e9

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
ring_buffer.adb:8:21: info: range check proved
ring_buffer.adb:13:61: info: range check proved, in call inlined at ring_buffer.
↳adb:26
ring_buffer.adb:15:16: info: discriminant check proved, in call inlined at ring_
↳buffer.adb:26
ring_buffer.adb:16:45: info: range check proved, in call inlined at ring_buffer.
↳adb:26
ring_buffer.adb:16:45: info: length check proved, in call inlined at ring_buffer.
↳adb:26
ring_buffer.adb:21:25: info: index check proved
ring_buffer.adb:23:37: info: index check proved
ring_buffer.adb:25:24: info: range check proved
ring_buffer.ads:10:07: info: range check proved
ring_buffer.ads:21:14: info: postcondition proved
```

Everything's fine here. Model is only accessed inside Update_Model, itself a ghost procedure, so it's fine to declare local variable New_Length without the Ghost aspect: everything inside a ghost procedure body is ghost. Moreover, we don't need to add any contract to Update_Model: it's inlined by GNATprove because it's a local procedure without a contract.

5.4.6 Example #6

The function Max_Array takes two arrays of the same length (but not necessarily with the same bounds) as arguments and returns an array with each entry being the maximum values of both arguments at that index.

Listing 44: array_util.ads

```
1 package Array_Util is
2
3   type Nat_Array is array (Positive range <>) of Natural;
4
5   function Max_Array (A, B : Nat_Array) return Nat_Array with
6     Pre => A'Length = B'Length;
7
8 end Array_Util;
```

Listing 45: array_util.adb

```
1 package body Array_Util is
2
3   function Max_Array (A, B : Nat_Array) return Nat_Array is
4     R : Nat_Array (A'Range);
5     J : Integer := B'First;
6   begin
7     for I in A'Range loop
8       if A (I) > B (J) then
9         R (I) := A (I);
10      else
11        R (I) := B (J);
12      end if;
```

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```

13     J := J + 1;
14     end loop;
15     return R;
16 end Max_Array;
17
18 end Array_Util;

```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Example_06
MD5: 4b8a6a9b1a3d4d228fe1e944914084fe

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
array_util.adb:8:24: medium: array index check might fail [reason for check: value_
↳must be a valid index into the array] [possible fix: loop at line 7 should_
↳mention J in a loop invariant]
array_util.adb:13:17: medium: overflow check might fail, cannot prove upper bound_
↳for J + 1 [reason for check: result of addition must fit in a 32-bits machine_
↳integer] [possible fix: loop at line 7 should mention J in a loop invariant]
gnatprove: unproved check messages considered as errors

```

This program is correct, but GNATprove can't prove that J is always in the index range of B (the unproved index check) or even that it's always within the bounds of its type (the unproved overflow check). Indeed, when checking the body of the loop, GNATprove forgets everything about the current value of J because it's been modified by previous loop iterations. To get more precise results, we need to provide a loop invariant.

5.4.7 Example #7

Let's add a loop invariant that states that J stays in the index range of B and let's protect the increment to J by checking that it's not already the maximal integer value.

Listing 46: array_util.ads

```

1 package Array_Util is
2
3     type Nat_Array is array (Positive range <>) of Natural;
4
5     function Max_Array (A, B : Nat_Array) return Nat_Array with
6         Pre => A'Length = B'Length;
7
8 end Array_Util;

```

Listing 47: array_util.adb

```

1 package body Array_Util is
2
3     function Max_Array (A, B : Nat_Array) return Nat_Array is
4         R : Nat_Array (A'Range);
5         J : Integer := B'First;
6     begin
7         for I in A'Range loop
8             pragma Loop_Invariant (J in B'Range);
9             if A (I) > B (J) then
10                R (I) := A (I);

```

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```

11     else
12         R (I) := B (J);
13     end if;
14     if J < Integer'Last then
15         J := J + 1;
16     end if;
17 end loop;
18 return R;
19 end Max_Array;
20
21 end Array_Util;

```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Example_07
MD5: 917629e0683725c23198f8a905a73c57

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
array_util.adb:8:33: medium: loop invariant might not be preserved by an arbitrary_
↳iteration
gnatprove: unproved check messages considered as errors

```

The loop invariant now allows verifying that no runtime error can occur in the loop's body (property INSIDE seen in section *Loop Invariants* (page 117)). Unfortunately, GNATprove fails to verify that the invariant stays valid after the first iteration of the loop (property PRESERVE). Indeed, knowing that J is in B'Range in a given iteration isn't enough to prove it'll remain so in the next iteration. We need a more precise invariant, linking J to the value of the loop index I, like $J = I - A'First + B'First$.

5.4.8 Example #8

We now consider a version of Max_Array which takes arguments that have the same bounds. We want to prove that Max_Array returns an array of the maximum values of both its arguments at each index.

Listing 48: array_util.ads

```

1 package Array_Util is
2
3     type Nat_Array is array (Positive range <>) of Natural;
4
5     function Max_Array (A, B : Nat_Array) return Nat_Array with
6         Pre => A'First = B'First and A'Last = B'Last,
7         Post => (for all K in A'Range =>
8             Max_Array'Result (K) = Natural'Max (A (K), B (K)));
9
10 end Array_Util;

```

Listing 49: array_util.adb

```

1 package body Array_Util is
2
3     function Max_Array (A, B : Nat_Array) return Nat_Array is
4         R : Nat_Array (A'Range) := (others => 0);
5     begin

```

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```

6     for I in A'Range loop
7         pragma Loop_Invariant (for all K in A'First .. I =>
8                                 R (K) = Natural'Max (A (K), B (K)));
9         if A (I) > B (I) then
10            R (I) := A (I);
11        else
12            R (I) := B (I);
13        end if;
14    end loop;
15    return R;
16 end Max_Array;
17
18 end Array_Util;

```

Listing 50: main.adb

```

1 with Array_Util; use Array_Util;
2
3 procedure Main is
4     A : Nat_Array := (1, 1, 2);
5     B : Nat_Array := (2, 1, 0);
6     R : Nat_Array (1 .. 3);
7 begin
8     R := Max_Array (A, B);
9 end Main;

```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Example_08
MD5: d0a04c214a632466a4fe4ec6cb7f8842

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
main.adb:8:09: medium: length check might fail [reason for check: array must be of
↳the appropriate length]
array_util.adb:8:35: medium: loop invariant might not be preserved by an arbitrary
↳iteration, cannot prove R (K) = Natural'max
array_util.adb:8:35: medium: loop invariant might fail in first iteration, cannot
↳prove R (K) = Natural'max
gnatprove: unproved check messages considered as errors

```

Runtime output

```

raised ADA.ASSERTIONS.ASSERTION_ERROR : Loop_Invariant failed at array_util.adb:7

```

Here, GNATprove doesn't manage to prove the loop invariant even for the first loop iteration (property INIT seen in section [Loop Invariants](#) (page 117)). In fact, the loop invariant is incorrect, as you can see by executing the function `Max_Array` with assertions enabled: at each loop iteration, `R` contains the maximum of `A` and `B` only until `I - 1` because the `I`'th index wasn't yet handled.

5.4.9 Example #9

We now consider a procedural version of `Max_Array` which updates its first argument instead of returning a new array. We want to prove that `Max_Array` sets the maximum values of both its arguments into each index in its first argument.

Listing 51: `array_util.ads`

```

1 package Array_Util is
2
3   type Nat_Array is array (Positive range <>) of Natural;
4
5   procedure Max_Array (A : in out Nat_Array; B : Nat_Array) with
6     Pre  => A'First = B'First and A'Last = B'Last,
7     Post => (for all K in A'Range =>
8       A (K) = Natural'Max (A'Old (K), B (K)));
9
10  end Array_Util;
```

Listing 52: `array_util.adb`

```

1 package body Array_Util is
2
3   procedure Max_Array (A : in out Nat_Array; B : Nat_Array) is
4     begin
5       for I in A'Range loop
6         pragma Loop_Invariant
7           (for all K in A'First .. I - 1 =>
8             A (K) = Natural'Max (A'Loop_Entry (K), B (K)));
9         pragma Loop_Invariant
10          (for all K in I .. A'Last => A (K) = A'Loop_Entry (K));
11         if A (I) <= B (I) then
12           A (I) := B (I);
13         end if;
14       end loop;
15     end Max_Array;
16
17  end Array_Util;
```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Example_09
MD5: 2de4bdd9c59d7d1eccb6259067ffdcf3

Prover output

```

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
array_util.adb:7:13: info: loop invariant preservation proved
array_util.adb:7:13: info: loop invariant initialization proved
array_util.adb:7:39: info: overflow check proved
array_util.adb:8:18: info: index check proved
array_util.adb:8:50: info: index check proved
array_util.adb:8:57: info: index check proved
array_util.adb:10:13: info: loop invariant initialization proved
array_util.adb:10:13: info: loop invariant preservation proved
array_util.adb:10:44: info: index check proved
array_util.adb:10:63: info: index check proved
array_util.adb:11:25: info: index check proved
array_util.adb:12:25: info: index check proved
array_util.ads:7:14: info: postcondition proved
array_util.ads:8:20: info: index check proved
```

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```
array_util.ads:8:45: info: index check proved
array_util.ads:8:52: info: index check proved
```

Everything is proved. The first loop invariant states that the values of A before the loop index contains the maximum values of the arguments of Max_Array (referring to the input value of A with A'Loop_Entry). The second loop invariant states that the values of A beyond and including the loop index are the same as they were on entry. This is the frame condition of the loop.

5.4.10 Example #10

Let's remove the frame condition from the previous example.

Listing 53: array_util.ads

```
1 package Array_Util is
2
3   type Nat_Array is array (Positive range <>) of Natural;
4
5   procedure Max_Array (A : in out Nat_Array; B : Nat_Array) with
6     Pre => A'First = B'First and A'Last = B'Last,
7     Post => (for all K in A'Range =>
8       A (K) = Natural'Max (A'Old (K), B (K)));
9
10  end Array_Util;
```

Listing 54: array_util.adb

```
1 package body Array_Util is
2
3   procedure Max_Array (A : in out Nat_Array; B : Nat_Array) is
4     begin
5       for I in A'Range loop
6         pragma Loop_Invariant
7           (for all K in A'First .. I - 1 =>
8             A (K) = Natural'Max (A'Loop_Entry (K), B (K)));
9         if A (I) <= B (I) then
10          A (I) := B (I);
11        end if;
12      end loop;
13    end Max_Array;
14
15  end Array_Util;
```

Code block metadata

```
Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Example_10
MD5: 8bdc8432cbb3f26f58f63457408c7172
```

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
array_util.adb:7:13: info: loop invariant initialization proved
array_util.adb:7:13: info: loop invariant preservation proved
array_util.adb:7:39: info: overflow check proved
array_util.adb:8:18: info: index check proved
array_util.adb:8:50: info: index check proved
```

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```
array_util.adb:8:57: info: index check proved
array_util.adb:9:25: info: index check proved
array_util.adb:10:25: info: index check proved
array_util.ads:7:14: info: postcondition proved
array_util.ads:8:20: info: index check proved
array_util.ads:8:45: info: index check proved
array_util.ads:8:52: info: index check proved
```

Everything is still proved. GNATprove internally generates the frame condition for the loop, so it's sufficient here to state that A before the loop index contains the maximum values of the arguments of `Max_Array`.