Introduction to SPARK

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Introduction to SPARK

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### CONTENTS:

1 SPARK Overview

1.1 What is it? ................................................................. 3
1.2 What do the tools do? .................................................. 4
1.3 Key Tools .......................................................................... 4
1.4 A trivial example ........................................................... 4
1.5 The Programming Language ............................................. 5
1.6 Limitations ................................................................. 5
  1.6.1 No side-effects in expressions ................................. 5
  1.6.2 No aliasing of names .............................................. 8
1.7 Designating SPARK Code ............................................... 10
1.8 Code Examples / Pitfalls ................................................ 11
  1.8.1 Example #1 ............................................................ 11
  1.8.2 Example #2 ........................................................... 12
  1.8.3 Example #3 ........................................................... 13
  1.8.4 Example #4 ........................................................... 14
  1.8.5 Example #5 ........................................................... 15
  1.8.6 Example #6 ........................................................... 16
  1.8.7 Example #7 ........................................................... 17
  1.8.8 Example #8 ........................................................... 17
  1.8.9 Example #9 ........................................................... 18
  1.8.10 Example #10 ....................................................... 20

2 Flow Analysis

2.1 What does flow analysis do? .......................................... 21
2.2 Errors Detected ............................................................ 21
  2.2.1 Uninitialized Variables .......................................... 21
  2.2.2 Ineffective Statements .......................................... 22
  2.2.3 Incorrect Parameter Mode .................................... 24
2.3 Additional Verifications ................................................ 25
  2.3.1 Global Contracts .................................................. 25
  2.3.2 Depends Contracts ............................................... 26
2.4 Shortcomings .............................................................. 28
  2.4.1 Modularity ........................................................... 28
  2.4.2 Composite Types .................................................. 29
  2.4.3 Value Dependency ................................................. 31
  2.4.4 Contract Computation .......................................... 33
2.5 Code Examples / Pitfalls ................................................ 33
  2.5.1 Example #1 ............................................................ 33
  2.5.2 Example #2 ........................................................... 34
  2.5.3 Example #3 ........................................................... 35
  2.5.4 Example #4 ........................................................... 36
  2.5.5 Example #5 ........................................................... 38
  2.5.6 Example #6 ........................................................... 39
  2.5.7 Example #7 ........................................................... 40
5.3 Guide Proof ..................................................... 112
  5.3.1 Local Ghost Variables ..................................... 112
  5.3.2 Ghost Procedures .......................................... 114
  5.3.3 Handling of Loops ......................................... 115
  5.3.4 Loop Invariants ............................................ 117
5.4 Code Examples / Pitfalls ....................................... 122
  5.4.1 Example #1 .................................................. 122
  5.4.2 Example #2 .................................................. 124
  5.4.3 Example #3 .................................................. 125
  5.4.4 Example #4 .................................................. 126
  5.4.5 Example #5 .................................................. 128
  5.4.6 Example #6 .................................................. 129
  5.4.7 Example #7 .................................................. 130
  5.4.8 Example #8 .................................................. 131
  5.4.9 Example #9 .................................................. 133
  5.4.10 Example #10 ............................................... 134
This tutorial is an interactive introduction to the SPARK programming language and its formal verification tools. You will learn the difference between Ada and SPARK and how to use the various analysis tools that come with SPARK.

This document was prepared by Claire Dross and Yannick Moy.

**Note:** The code examples in this course use an 80-column limit, which is a typical limit for Ada code. Note that, on devices with a small screen size, some code examples might be difficult to read.

**Note:** Each code example from this book has an associated "code block metadata", which contains the name of the "project" and an MD5 hash value. This information is used to identify a single code example.

You can find all code examples in a zip file, which you can download from the learn website. The directory structure in the zip file is based on the code block metadata. For example, if you're searching for a code example with this metadata:

- Project: Courses.Intro_To_Ada.Imperative.Language.Greet
- MD5: cba89a34b87c9dafa71533d982d05e6ab

you will find it in this directory:

`projects/Courses/Intro_To_Ada/Imperative.Language/Greet/cba89a34b87c9dafa71533d982d05e6ab/`

In order to use this code example, just follow these steps:

1. Unpack the zip file;
2. Go to target directory;
3. Start GNAT Studio on this directory;
4. Build (or compile) the project;
5. Run the application (if a main procedure is available in the project).

---

1. [http://creativecommons.org/licenses/by-sa/4.0](http://creativecommons.org/licenses/by-sa/4.0)
This tutorial is an introduction to the SPARK programming language and its formal verification tools. You need not know any specific programming language (although going over the Introduction to Ada course first may help) or have experience in formal verification.

1.1 What is it?

SPARK refers to two different things:

- a programming language targeted at functional specification and static verification, and
- a set of development and verification tools for that language.

The SPARK language is based on a subset of the Ada language. Ada is particularly well suited to formal verification since it was designed for critical software development. SPARK builds on that foundation.

Version 2012 of Ada introduced the use of aspects, which can be used for subprogram contracts, and version 2014 of SPARK added its own aspects to further aid static analysis.
1.2 What do the tools do?

We start by reviewing static verification of programs, which is verification of the source code performed without compiling or executing it. Verification uses tools that perform static analysis. These can take various forms. They include tools that check types and enforce visibility rules, such as the compiler, in addition to those that perform more complex reasoning, such as abstract interpretation, as done by a tool like CodePeer from AdaCore. The tools that come with SPARK perform two different forms of static analysis:

- **flow analysis** is the fastest form of analysis. It checks initializations of variables and looks at data dependencies between inputs and outputs of subprograms. It can also find unused assignments and unmodified variables.
- **proof** checks for the absence of runtime errors as well as the conformance of the program with its specifications.

1.3 Key Tools

The tool for formal verification of the SPARK language is called *GNATprove*. It checks for conformance with the SPARK subset and performs flow analysis and proof of the source code. Several other tools support the SPARK language, including both the GNAT compiler and the GNAT Studio integrated development environment.

1.4 A trivial example

We start with a simple example of a subprogram in Ada that uses SPARK aspects to specify verifiable subprogram contracts. The subprogram, called Increment, adds 1 to the value of its parameter X:

Listing 1: increment.ads

```
procedure Increment
  (X : in out Integer)
with
  Global => null,
  Depends => (X => X),
  Pre => X < Integer'Last,
  Post => X = X'Old + 1;
```

Listing 2: increment.adb

```
procedure Increment
  (X : in out Integer)
is
  begin
    X := X + 1;
  end Increment;
```

Code block metadata

Project: Courses.Intro_To_Spark.Overview.Trivial_Example
MD5: ce28b1facb44917b6cc208639c187064

3 https://www.adacore.com/codepeer
4 https://www.adacore.com/gnatpro
5 https://www.adacore.com/gnatpro/toolsuite/gps
Prover output

Phase 1 of 2: generation of Global contracts ...  
Phase 2 of 2: flow analysis and proof ...  
increment.adb:5:10: info: overflow check proved  
increment.ads:4:03: info: data dependencies proved  
increment.ads:5:03: info: flow dependencies proved  
increment.ads:7:14: info: postcondition proved  
increment.ads:7:24: info: overflow check proved

The contracts are written using the Ada aspect feature and those shown specify several properties of this subprogram:

- The SPARK Global aspect says that Increment does not read or write any global variables.
- The SPARK Depend aspect is especially interesting for security: it says that the value of the parameter X after the call depends only on the (previous) value of X.
- The Pre and Post aspects of Ada specify functional properties of Increment:
  - Increment is only allowed to be called if the value of X prior to the call is less than Integer'Last. This ensures that the addition operation performed in the subprogram body doesn't overflow.
  - Increment does indeed perform an increment of X: the value of X after a call is one greater than its value before the call.

GNATprove can verify all of these contracts. In addition, it verifies that no error can be raised at runtime when executing Increment's body.

1.5 The Programming Language

It's important to understand why there are differences between the SPARK and Ada languages. The aim when designing the SPARK subset of Ada was to create the largest possible subset of Ada that was still amenable to simple specification and sound verification.

The most notable restrictions from Ada are related to exceptions and access types, both of which are known to considerably increase the amount of user-written annotations required for full support. Backwards goto statements and controlled types are also not supported since they introduce non-trivial control flow. The two remaining restrictions relate to side-effects in expressions and aliasing of names, which we now cover in more detail.

1.6 Limitations

1.6.1 No side-effects in expressions

The SPARK language doesn't allow side-effects in expressions. In other words, evaluating a SPARK expression must not update any object. This limitation is necessary to avoid unpredictable behavior that depends on order of evaluation, parameter passing mechanisms, or compiler optimizations. The expression for Dummy below is non-deterministic due to the order in which the two calls to F are evaluated. It's therefore not legal SPARK.
Listing 3: show_illegal_ada_code.adb

```ada
procedure Show_Illegal_Ada_Code is
  function F (X : in out Integer) return Integer is
    Tmp : constant Integer := X;
    begin
    X := X + 1;
    return Tmp;
  end F;
  Dummy : Integer := 0;
  begin
  Dummy := F (Dummy) - F (Dummy); -- ??
end Show_Illegal_Ada_Code;
```

**Code block metadata**

*Project: Courses.Intro_To_Spark.Overview.Illegal_Ada_Code*

*MD5: a5cbf1824526857da94791ac1790200c*

**Build output**

show_illegal_ada_code.adb:13:28: error: value may be affected by call to "F"
  because order of evaluation is arbitrary

gprbuild: *** compilation phase failed

**Prover output**

Phase 1 of 2: generation of Global contracts ...
show_illegal_ada_code.adb:13:28: error: value may be affected by call to "F"
  because order of evaluation is arbitrary
gnatprove: error during generation of Global contracts

In fact, the code above is not even legal Ada, so the same error is generated by the GNAT compiler. But SPARK goes further and GNATprove also produces an error for the following equivalent code that is accepted by the Ada compiler:

Listing 4: show_illegal_spark_code.adb

```ada
procedure Show_Illegal_SPARK_Code is
  Dummy : Integer := 0;
  function F return Integer is
    Tmp : constant Integer := Dummy;
    begin
    Dummy := Dummy + 1;
    return Tmp;
  end F;
  begin
  Dummy := F - F; -- ??
end Show_Illegal_SPARK_Code;
```

**Code block metadata**

*Project: Courses.Intro_To_Spark.Overview.Illegal_SPARK_Code*

*MD5: e747edb6ee147adb7fba97c9e7c8d5ef*

**Prover output**
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_illegal_spark_code.adb:5:13: error: function with output global "Dummy" is not allowed in SPARK
gnatprove: error during analysis of data and information flow

The SPARK language enforces the lack of side-effects in expressions by forbidding side-effects in functions, which include modifications to either parameters or global variables. As a consequence, SPARK forbids functions with \texttt{out} or \texttt{in out} parameters in addition to functions modifying a global variable. Function \texttt{F} below is illegal in SPARK, while Function \texttt{Incr} might be legal if it doesn't modify any global variables and function \texttt{Incr\_And\_Log} might be illegal if it modifies global variables to perform logging.

\begin{verbatim}
function F (X : in out Integer) return Integer; -- Illegal
function Incr (X : Integer) return Integer; -- OK?
function Incr\_And\_Log (X : Integer) return Integer; -- OK?
\end{verbatim}

In most cases, you can easily replace these functions by procedures with an \texttt{out} parameter that returns the computed value.

When it has access to function bodies, GNATprove verifies that those functions are indeed free from side-effects. Here for example, the two functions \texttt{Incr} and \texttt{Incr\_And\_Log} have the same signature, but only \texttt{Incr} is legal in SPARK. \texttt{Incr\_And\_Log} isn't: it attempts to update the global variable \texttt{Call\_Count}.

\begin{verbatim}
Listing 5: side\_effects.ads

package Side\_Effects is

  function Incr (X : Integer) return Integer; -- OK?
  function Incr\_And\_Log (X : Integer) return Integer; -- OK?

end Side\_Effects;

Listing 6: side\_effects.adb

package body Side\_Effects is

  function Incr (X : Integer) return Integer
  is (X + 1); -- OK

  Call\_Count : Natural := 0;

  function Incr\_And\_Log (X : Integer) return Integer is
begin
  Call\_Count := Call\_Count + 1; -- Illegal
  return X + 1;
end Incr\_And\_Log;

end Side\_Effects;
\end{verbatim}

\textbf{Code block metadata}

Project: Courses.Intro\_To\_Spark.Overview.Side\_Effects
MD5: 1b555e4b7bb519eea4df718a9356a2ed

\textbf{Prover output}

\section*{1.6. Limitations}
Introduction to SPARK

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
side_effects.ads:5:13: error: function with output global "Call_Count" is not allowed in SPARK
gnatprove: error during analysis of data and information flow

1.6.2 No aliasing of names

Another restriction imposed by the SPARK subset concerns aliasing\(^6\). We say that two names are aliased if they refer to the same object. There are two reasons why aliasing is forbidden in SPARK:

- It makes verification more difficult because it requires taking into account the fact that modifications to variables with different names may actually update the same object.
- Results may seem unexpected from a user point of view. The results of a subprogram call may depend on compiler-specific attributes, such as parameter passing mechanisms, when its parameters are aliased.

Aliasing can occur as part of the parameter passing that occurs in a subprogram call. Functions have no side-effects in SPARK, so aliasing of parameters in function calls isn't problematic; we need only consider procedure calls. When a procedure is called, SPARK verifies that no \texttt{out} or \texttt{in} \texttt{out} parameter is aliased with either another parameter of the procedure or a global variable modified in the procedure's body.

Procedure \texttt{Move\_To\_Total} is an example where the possibility of aliasing wasn't taken into account by the programmer:

Listing 7: no_aliasing.adb

```
procedure No_Aliasing is
  Total : Natural := 0;
  procedure Move_To_Total (Source : in out Natural) with Post => Total = Total'Old + Source'Old and Source = 0 is
    begin
    Total := Total + Source;
    Source := 0;
    end Move_To_Total;
  X : Natural := 3;
begin
  Move_To_Total (X);
  pragma Assert (Total = 3); -- OK
  Move_To_Total (Total); -- flow analysis error
  pragma Assert (Total = 6); -- runtime error
end No_Aliasing;
```

Code block metadata

Project: Courses.Intro_To_Spark.Overview.Aliasing
MD5: 91038ef030fe27e3b000ab3db9c134ad

Prover output

\(^6\) https://en.wikipedia.org/wiki/Aliasing\_computing
Introduction to SPARK

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...

no_aliasing.adb:18:19: high: formal parameter "Source" and global "Total" are aliased (SPARK RM 6.4.2)
gnatprove: unproved check messages considered as errors

Runtime output

raised ADAASSERTIONSASSERTION_ERROR : no_aliasing.adb:19

Move_To_Total adds the value of its input parameter Source to the global variable Total and then resets Source to 0. The programmer has clearly not taken into account the possibility of an aliasing between Total and Source. (This sort of error is quite common.)

This procedure itself is valid SPARK. When doing verification, GNATprove assumes, like the programmer did, that there's no aliasing between Total and Source. To ensure this assumption is valid, GNATprove checks for possible aliasing on every call to Move_To_Total. Its final call in procedure No_Aliasing violates this assumption, which produces both a message from GNATprove and a runtime error (an assertion violation corresponding to the expected change in Total from calling Move_To_Total). Note that the postcondition of Move_To_Total is not violated on this second call since integer parameters are passed by copy and the postcondition is checked before the copy-back from the formal parameters to the actual arguments.

Aliasing can also occur as a result of using access types (pointers in Ada). These are restricted in SPARK so that only benign aliasing is allowed, when both names are only used to read the data. In particular, assignment between access objects operates a transfer of ownership, where the source object loses its permission to read or write the underlying allocated memory.

Procedure Ownership_Transfer is an example of code that is legal in Ada but rejected in SPARK due to aliasing:

Listing 8: ownership_transfer.adb

```
procedure Ownership_Transfer is
  type Int_Ptr is access Integer;
  X : Int_Ptr;
  Y : Int_Ptr;
  Dummy : Integer;
begin
  X := new Integer'(:1);
  X.all := X.all + 1;
  Y := X;
  Y.all := Y.all + 1;
  X.all := X.all + 1; -- illegal
  X.all := 1; -- illegal
  Dummy := X.all; -- illegal
end Ownership_Transfer;
```

Code block metadata

Project: Courses.Intro_To_Spark.Overview.Ownership_Transfer
MD5: 951fe1c930d43a5009e607994ae8dd03

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...

(continues on next page)

1.6. Limitations 9

7 https://en.wikipedia.org/wiki/Pointer_(computer_programming)
Introduction to SPARK

(continued from previous page)

ownership_transfer.adb:11:06: error: dereference from "X" is not writable
ownership_transfer.adb:11:06: error: object was moved at line 9
ownership_transfer.adb:11:15: error: dereference from "X" is not readable
ownership_transfer.adb:11:15: error: object was moved at line 9
ownership_transfer.adb:12:06: error: dereference from "X" is not writable
ownership_transfer.adb:12:06: error: object was moved at line 9
ownership_transfer.adb:13:15: error: dereference from "X" is not readable
ownership_transfer.adb:13:15: error: object was moved at line 9
gnatprove: error during analysis of data and information flow

After the assignment of X to Y, variable X cannot be used anymore to read or write the underlying allocated memory.

Note: For more details on these limitations, see the SPARK User's Guide.

1.7 Designating SPARK Code

Since the SPARK language is restricted to only allow easily specifiable and verifiable constructs, there are times when you can't or don't want to abide by these limitations over your entire code base. Therefore, the SPARK tools only check conformance to the SPARK subset on code which you identify as being in SPARK.

You do this by using an aspect named SPARK_Mode. If you don't explicitly specify otherwise, SPARK_Mode is Off, meaning you can use the complete set of Ada features in that code and that it should not be analyzed by GNATprove. You can change this default either selectively (on some units or subprograms or packages inside units) or globally (using a configuration pragma, which is what we're doing in this tutorial). To allow simple reuse of existing Ada libraries, entities declared in imported units with no explicit SPARK_Mode can still be used from SPARK code. The tool only checks for SPARK conformance on the declaration of those entities which are actually used within the SPARK code.

Here's a common case of using the SPARK_Mode aspect:

```ada
package P with SPARK_Mode => On is
  -- package spec is IN SPARK, so can be used by SPARK clients
end P;

package body P with SPARK_Mode => Off is
  -- body is NOT IN SPARK, so is ignored by GNATprove
end P;
```

The package P only defines entities whose specifications are in the SPARK subset. However, it wants to use all Ada features in its body. Therefore the body should not be analyzed and has its SPARK_Mode aspect set to Off.

You can specify SPARK_Mode in a fine-grained manner on a per-unit basis. An Ada package has four different components: the visible and private parts of its specification and the declarative and statement parts of its body. You can specify SPARK_Mode as being either On or Off on any of those parts. Likewise, a subprogram has two parts: its specification and its body.

---

Note: For more details on these limitations, see the SPARK User's Guide.

---

Introduction to SPARK

A general rule in SPARK is that once SPARK_Mode has been set to Off, it can never be switched On again in the same part of a package or subprogram. This prevents setting SPARK_Mode to On for local units of a unit with SPARK_Mode Off and switching back to SPARK_Mode On for a part of a given unit where it was set to Off in a previous part.

Note: For more details on the use of SPARK_Mode, see the SPARK User's Guide.

1.8 Code Examples / Pitfalls

1.8.1 Example #1

Here’s a package defining an abstract stack type (defined as a private type in SPARK) of Element objects along with some subprograms providing the usual functionalities of stacks. It’s marked as being in the SPARK subset.

Listing 9: stack_package.ads

```plaintext
package Stack_Package with SPARK_Mode => On is
type Element is new Natural;
type Stack is private;

function Empty return Stack;
procedure Push (S : in out Stack; E : Element);
function Pop (S : in out Stack) return Element;

private

type Stack is record
  Top : Integer;
  -- ...
end record;

end Stack_Package;
```

Code block metadata

Project: Courses.Intro_To_Spark.Overview.Example_01
MD5: 2b15e13e05435fb93406054d705b51c6

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
stack_package.ads:9:13: error: function with "in out" parameter is not allowed in SPARK
stack_package.ads:9:13: error: violation of aspect SPARK_Mode at line 2
gnatprove: error during analysis of data and information flow

Side-effects in expressions are not allowed in SPARK. Therefore, Pop is not allowed to modify its parameter S.

1.8.2 Example #2

Let's turn to an abstract state machine version of a stack, where the unit provides a single instance of a stack. The content of the stack (global variables Content and Top) is not directly visible to clients. In this stripped-down version, only the function Pop is available to clients. The package spec and body are marked as being in the SPARK subset.

Listing 10: global_stack.ads

```ada
package Global_Stack
  with SPARK_Mode => On
is
  type Element is new Integer;
  function Pop return Element;
end Global_Stack;
```

Listing 11: global_stack.adb

```ada
package body Global_Stack
  with SPARK_Mode => On
is
  Max : constant Natural := 100;
  type Element_Array is array (1 .. Max) of Element;
  Content : Element_Array;
  Top : Natural;

  function Pop return Element is
    E : constant Element := Content (Top);
  begin
    Top := Top - 1;
    return E;
  end Pop;
end Global_Stack;
```

Code block metadata

- **Project**: Courses.Intro_To_Spark.Overview.Example_02
- **MD5**: 8c4eb564643eeef48264b5e43a6f580b9

Prover output

- Phase 1 of 2: generation of Global contracts ...
- Phase 2 of 2: analysis of data and information flow ...
- global_stack.adb:7:04: warning: variable "Content" is read but not assigned [gnatwv]
- global_stack.ads:6:13: error: function with output global "Top" is not allowed in SPARK
- gnatprove: error during analysis of data and information flow

As above, functions should be free from side-effects. Here, Pop updates the global variable Top, which is not allowed in SPARK.
1.8.3 Example #3

We now consider two procedures: Permute and Swap. Permute applies a circular permutation to the value of its three parameters. Swap then uses Permute to swap the value of \( X \) and \( Y \).

Listing 12: p.ads

```ada
package P
  with SPARK_Mode => On
is
  procedure Permute (X, Y, Z : in out Positive);
  procedure Swap (X, Y : in out Positive);
end P;
```

Listing 13: p.adb

```ada
package body P
  with SPARK_Mode => On
is
  procedure Permute (X, Y, Z : in out Positive) is
    Tmp : constant Positive := X;
    begin
    X := Y;
    Y := Z;
    Z := Tmp;
    end Permute;

  procedure Swap (X, Y : in out Positive) is
    begin
    Permute (X, Y, Y);
    end Swap;
end P;
```

Listing 14: test_swap.adb

```ada
with P; use P;

procedure Test_Swap
  with SPARK_Mode => On
is
  A : Integer := 1;
  B : Integer := 2;
  begin
  Swap (A, B);
end Test_Swap;
```

Code block metadata

Project: Courses.Intro_To_Spark.Overview.Example_03
MD5: 0868a806061d86af4d2a03b1e7dc83c2

Build output

p.adb:14:19: error: writable actual for "Y" overlaps with actual for "Z"
gprbuild: *** compilation phase failed

Prover output

Phase 1 of 2: generation of Global contracts ...
p.adb:14:19: error: writable actual for "Y" overlaps with actual for "Z"
gnatprove: error during generation of Global contracts
Here, the values for parameters Y and Z are aliased in the call to Permute, which is not allowed in SPARK. In fact, in this particular case, this is even a violation of Ada rules so the same error is issued by the Ada compiler.

In this example, we see the reason why aliasing is not allowed in SPARK: since Y and Z are Positive, they are passed by copy and the result of the call to Permute depends on the order in which they're copied back after the call.

### 1.8.4 Example #4

Here, the Swap procedure is used to swap the value of the two record components of R.

**Listing 15: p.ads**

```ada
package P
  with SPARK_Mode => On
is
  type Rec is record
    F1 : Positive;
    F2 : Positive;
  end record;

  procedure Swap_Fields (R : in out Rec);
  procedure Swap (X, Y : in out Positive);
end P;
```

**Listing 16: p.adb**

```ada
package body P
  with SPARK_Mode => On
is
  procedure Swap (X, Y : in out Positive) is
    Tmp : constant Positive := X;
  begin
    X := Y;
    Y := Tmp;
  end Swap;

  procedure Swap_Fields (R : in out Rec) is
  begin
    Swap (R.F1, R.F2);
  end Swap_Fields;
end P;
```

**Prover output**

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...

This code is correct. The call to Swap is safe: two different components of the same record can't refer to the same object.
1.8.5 Example #5

Here’s a slight modification of the previous example using an array instead of a record: Swap_Indexes calls Swap on values stored in the array A.

Listing 17: p.ads

```ada
package P
  with SPARK_Mode => On
is
  type P_Array is array (Natural range <>) of Positive;
  procedure Swap_Indexes (A : in out P_Array; I, J : Natural);
  procedure Swap (X, Y : in out Positive);
end P;
```

Listing 18: p.adb

```ada
package body P
  with SPARK_Mode => On
is
  procedure Swap (X, Y : in out Positive) is
    Tmp : constant Positive := X;
  begin
    X := Y;
    Y := Tmp;
  end Swap;

  procedure Swap_Indexes (A : in out P_Array; I, J : Natural) is
  begin
    Swap (A (I), A (J));
  end Swap_Indexes;
end P;
```

Code block metadata

Project: Courses.Intro_To_Spark.Overview.Example_05
MD5: 62a9579572e36443995ff5a2d5ef08

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
p.adb:13:13: medium: formal parameters "X" and "Y" might be aliased (SPARK RM 6.4.2)
gnatprove: unproved check messages considered as errors

GNATprove detects a possible case of aliasing. Unlike the previous example, it has no way of knowing that the two elements A (I) and A (J) are actually distinct when we call Swap. GNATprove issues a check message here instead of an error, giving you the possibility of justifying the message after review (meaning that you’ve verified manually that this can’t, in fact, occur).
1.8.6 Example #6

We now consider a package declaring a type Dictionary, an array containing a word per letter. The procedure Store allows us to insert a word at the correct index in a dictionary.

Listing 19: p.ads

```ada
with Ada.Finalization;

package P with SPARK_Mode => On is
  subtype Letter is Character range 'a' .. 'z';
  type String_Access is new Ada.Finalization.Controlled with record
    Ptr : access String;
  end record;
  type Dictionary is array (Letter) of String_Access;

  procedure Store (D : in out Dictionary; W : String);
end P;
```

Listing 20: p.adb

```ada
package body P with SPARK_Mode => On is

  procedure Store (D : in out Dictionary; W : String) is
    First_Letter : constant Letter := W (W'First);
    begin
      D (First_Letter).Ptr := new String' (W);
    end Store;
  end P;
```

Code block metadata

Project: Courses.Intro_To_Spark.Overview.Example_06
MD5: 9175bcd1474e2143462b860c01d8602e

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
p.adb:7:07: error: "String_Access" is not allowed in SPARK (due to controlled types)
p.adb:7:07: error: violation of aspect SPARK_Mode at line 2
p.adb:7:31: error: borrow or observe of an expression which is not part of stand-alone object or parameter is not allowed in SPARK (SPARK RM 3.10(3))
p.adb:7:09: error: violation of aspect SPARK_Mode at line 2
p.adb:7:09: error: "Controlled" is not allowed in SPARK (due to controlled types)
p.adb:10:04: error: "String_Access" is not allowed in SPARK (due to controlled types)
p.adb:10:04: error: violation of aspect SPARK_Mode at line 4
gnatprove: error during analysis of data and information flow

This code is not correct: controlled types are not part of the SPARK subset. The solution here is to use SPARK_Mode to separate the definition of String_Access from the rest of the code in a fine grained manner.
1.8.7 Example #7

Here's a new version of the previous example, which we've modified to hide the controlled type inside the private part of package P, using pragma SPARK_Mode (Off) at the start of the private part.

Listing 21: p.ads

```ada
with Ada.Finalization;

package P
  with SPARK_Mode => On
is
  subtype Letter is Character range 'a' .. 'z';
  type String_Access is private;
  type Dictionary is array (Letter) of String_Access;

  function New_String_Access (W : String) return String_Access;
  procedure Store (D : in out Dictionary; W : String);

private
  pragma SPARK_Mode (Off);

  type String_Access is new Ada.Finalization.Controlled with record
    Ptr : access String;
  end record;

  function New_String_Access (W : String) return String_Access is
    (Ada.Finalization.Controlled with Ptr => new String'(W));
end P;
```

Code block metadata

Project: Courses.Intro_To_Spark.Overview.Example_07
MD5: cb04206c9734eb95f6444757d005dae2

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...

Since the controlled type is defined and used inside of a part of the code ignored by GNATprove, this code is correct.

1.8.8 Example #8

Let's put together the new spec for package P with the body of P seen previously.

Listing 22: p.ads

```ada
with Ada.Finalization;

package P
  with SPARK_Mode => On
is
  subtype Letter is Character range 'a' .. 'z';
  type String_Access is private;
  type Dictionary is array (Letter) of String_Access;
```

(continues on next page)
function New_String_Access (W : String) return String_Access;

procedure Store (D : in out Dictionary; W : String);

private
pragma SPARK_Mode (Off);

type String_Access is new Ada.Finalization.Controlled with record
  Ptr : access String;
end record;

function New_String_Access (W : String) return String_Access is
  (Ada.Finalization.Controlled with Ptr => new String' (W));
end P;

Listing 23: p.adb

package body P
  with SPARK_Mode => On
is
  procedure Store (D : in out Dictionary; W : String) is
    First_Letter : constant Letter := W (W'First);
    begin
      D (First_Letter) := New_String_Access (W);
    end Store;
  end P;

1.8.9 Example #9

Next, we moved the declaration and the body of the procedure Store to another package named Q.

Listing 24: p.ads

with Ada.Finalization;

package P
  with SPARK_Mode => On
is
(continues on next page)
subtype Letter is Character range 'a' .. 'z';
type String_Access is private;
type Dictionary is array (Letter) of String_Access;

function New_String_Access (W : String) return String_Access is
  (Ada.Finalization.Controlled with Ptr => new String' (W));
end P;

Listing 25: q.ads

with P; use P;
package Q
  with SPARK_Mode => On
is
  procedure Store (D : in out Dictionary; W : String);
end Q;

Listing 26: q.adb

package body Q
  with SPARK_Mode => On
is
  procedure Store (D : in out Dictionary; W : String) is
    First_Letter : constant Letter := W (W'First);
  begin
    D (First_Letter) := New_String_Access (W);
  end Store;
end Q;

And now everything is fine: we've managed to retain the use of the controlled type while having most of our code in the SPARK subset so GNATprove is able to analyze it.
1.8.10 Example #10

Our final example is a package with two functions to search for the value 0 inside an array \( A \). The first raises an exception if 0 isn't found in \( A \) while the other simply returns 0 in that case.

Listing 27: p.ads

```
package P
  with SPARK_Mode => On
is
  type N_Array is array (Positive range <>) of Natural;
  Not_Found : exception;

  function Search_Zero_P (A : N_Array) return Positive;
  function Search_Zero_N (A : N_Array) return Natural;
end P;
```

Listing 28: p.adb

```
package body P
  with SPARK_Mode => On
is
  function Search_Zero_P (A : N_Array) return Positive is
    begin
      for I in A'Range loop
        if A (I) = 0 then
          return I;
        end if;
      end loop;
      raise Not_Found;
    end Search_Zero_P;

  function Search_Zero_N (A : N_Array) return Natural
    with SPARK_Mode => Off is
    begin
      return Search_Zero_P (A);
    exception
      when Not_Found => return 0;
    end Search_Zero_N;
end P;
```

Code block metadata

- Project: Courses.Intro_To_Spark.Overview.Example_10
- MD5: 4b9656690ab1d42cebc728177f8a00637

Prover output

- Phase 1 of 2: generation of Global contracts ...
- Phase 2 of 2: flow analysis and proof ...
- p.adb:11:07: medium: exception might be raised
- gnatprove: unproved check messages considered as errors

This code is perfectly correct, despite the use of exception handling, because we've carefully isolated this non-SPARK feature in a function body marked with a SPARK_Mode of Off so it's ignored by GNATprove. However, GNATprove tries to show that Not_Found is never raised in Search_Zero_P, producing a message about a possible exception being raised. Looking at Search_Zero_N, it's indeed likely that an exception is meant to be raised in some cases, which means you need to verify that Not_Found is only raised when appropriate using other methods such as peer review or testing.
In this section we present the flow analysis capability provided by the GNATprove tool, a critical tool for using SPARK.

2.1 What does flow analysis do?

Flow analysis concentrates primarily on variables. It models how information flows through them during a subprogram's execution, connecting the final values of variables to their initial values. It analyzes global variables declared at library level, local variables, and formal parameters of subprograms.

Nesting of subprograms creates what we call scope variables: variables declared locally to an enclosing unit. From the perspective of a nested subprogram, scope variables look very much like global variables.

Flow analysis is usually fast, roughly as fast as compilation. It detects various types of errors and finds violations of some SPARK legality rules, such as the absence of aliasing and freedom of expressions from side-effects. We discussed these rules in the SPARK Overview (page 3).

Flow analysis is sound: if it doesn't detect any errors of a type it's supposed to detect, we know for sure there are no such errors.

2.2 Errors Detected

2.2.1 Uninitialized Variables

We now present each class of errors detected by flow analysis. The first is the reading of an uninitialized variable. This is nearly always an error: it introduces non-determinism and breaks the type system because the value of an uninitialized variable may be outside the range of its subtype. For these reasons, SPARK requires every variable to be initialized before being read.

Flow analysis is responsible for ensuring that SPARK code always fulfills this requirement. For example, in the function Max_Array shown below, we've neglected to initialize the value of Max prior to entering the loop. As a consequence, the value read by the condition of the if statement may be uninitialized. Flow analysis detects and reports this error.

Listing 1: show_uninitialized.ads

```ada
package Show_Uninitialized is
  type Array_Of_Naturals is array (Integer range <>) of Natural;
```

(continues on next page)
Ineffective Statements

Ineffective statements are different than dead code: they're executed, and often even modify the value of variables, but have no effect on any of the subprogram's visible outputs: parameters, global variables or the function result. Ineffective statements should be avoided because they make the code less readable and more difficult to maintain.

More importantly, they're often caused by errors in the program: the statement may have been written for some purpose, but isn't accomplishing that purpose. These kinds of errors can be difficult to detect in other ways.

For example, the subprograms Swap1 and Swap2 shown below don't properly swap their two parameters X and Y. This error caused a statement to be ineffective. That ineffective statement is not an error in itself, but flow analysis produces a warning since it can be indicative of an error, as it is here.

So far, we've seen examples where flow analysis warns about ineffective statements and unused variables.
2.2.3 Incorrect Parameter Mode

Parameter modes are an important part of documenting the usage of a subprogram and affect the code generated for that subprogram. Flow analysis checks that each specified parameter mode corresponds to the usage of that parameter in the subprogram's body. It checks that an \texttt{in} parameter is never modified, either directly or through a subprogram call, checks that the initial value of an \texttt{out} parameter is never read in the subprogram (since it may not be defined on subprogram entry), and warns when an \texttt{in out} parameter isn't modified or when its initial value isn't used. All of these may be signs of an error.

We see an example below. The subprogram \texttt{Swap} is incorrect and GNATprove warns about an input which isn't read:

```
package Show_Incorrect_Param_Mode is
  type T is new Integer;
  procedure Swap (X, Y : in out T);
end Show_Incorrect_Param_Mode;
```

```
package body Show_Incorrect_Param_Mode is
  procedure Swap (X, Y : in out T) is
    Tmp : T := X;
  begin
    Y := X;  -- The initial value of Y is not used
    X := Tmp; -- Y is computed to be an out parameter
  end Swap;
end Show_Incorrect_Param_Mode;
```

In SPARK, unlike Ada, you should declare an \texttt{out} parameter to be \texttt{in out} if it's not modified on every path, in which case its value may depend on its initial value. SPARK is stricter than Ada to allow more static detection of errors. This table summarizes SPARK's valid parameter modes as a function of whether reads and writes are done to the parameter.

<table>
<thead>
<tr>
<th>Initial value read</th>
<th>Written on some path</th>
<th>Written on every path</th>
<th>Parameter mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>X</td>
<td></td>
<td>\texttt{in}</td>
</tr>
<tr>
<td>X</td>
<td>X</td>
<td></td>
<td>\texttt{in out}</td>
</tr>
<tr>
<td>X</td>
<td>X</td>
<td></td>
<td>\texttt{in out}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>X</td>
<td>\texttt{in out}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>X</td>
<td>\texttt{out}</td>
</tr>
</tbody>
</table>
2.3 Additional Verifications

2.3.1 Global Contracts

So far, none of the verifications we've seen require you to write any additional annotations. However, flow analysis also checks flow annotations that you write. In SPARK, you can specify the set of global and scoped variables accessed or modified by a subprogram. You do this using a contract named Global.

When you specify a Global contract for a subprogram, flow analysis checks that it's both correct and complete, meaning that no variables other than those stated in the contract are accessed or modified, either directly or through a subprogram call, and that all those listed are accessed or modified. For example, we may want to specify that the function Get_Value_Of_X reads the value of the global variable X and doesn't access any other global variable. If we do this through a comment, as is usually done in other languages, GNATprove can't verify that the code complies with this specification:

```plaintext
package Show_Global_Contracts is
  X, Y, Z : Natural := 0;
  function Get_Value_Of_X return Natural;
    -- Get_Value_Of_X reads the value of the global variable X
end Show_Global_Contracts;
```

You write global contracts as part of the subprogram specification. In addition to their value in flow analysis, they also provide useful information to users of a subprogram. The value you specify for the Global aspect is an aggregate-like list of global variable names, grouped together according to their mode.

In the example below, the procedure Set_X_TO_Y_Plus_Z reads both Y and Z. We indicate this by specifying them as the value for Input. It also writes X, which we specify using Output. Since Set_X_TO_X_Plus_Y both writes X and reads its initial value, X's mode is In_Out. Like parameters, if no mode is specified in a Global aspect, the default is Input. We see this in the case of the declaration of Get_Value_Of_X. Finally, if a subprogram, such as Incr_Parameter_X, doesn't reference any global variables, you set the value of the global contract to null.

Listing 7: show_global_contracts.ads

```plaintext
package Show_Global_Contracts is
  X, Y, Z : Natural := 0;
  function Get_Value_Of_X return Natural;
    -- Get_Value_Of_X reads the value of the global variable X
end Show_Global_Contracts;

procedure Set_X_TO_Y_Plus_Z with
  Global => (Input => (Y, Z), -- reads values of Y and Z
             Output => X); -- modifies value of X

procedure Set_X_TO_X_Plus_Y with
  Global => (Input => Y, -- reads value of Y
             In_Out => X); -- modifies value of X and
                 -- also reads its initial value

function Get_Value_Of_X return Natural with
  Global => X; -- reads the value of the global variable X

procedure Incr_Parameter_X (X : in out Natural) with
  Global => null; -- do not reference any global variable
```

(continues on next page)
2.3.2 Depends Contracts

You may also supply a Depends contract for a subprogram to specify dependencies between its inputs and outputs. These dependencies include not only global variables but also parameters and the function's result. When you supply a Depends contract for a subprogram, flow analysis checks that it's correct and complete, that is, for each dependency you list, the variable depends on those listed and on no others.

For example, you may want to say that the new value of each parameter of Swap, shown below, depends only on the initial value of the other parameter and that the value of X after the return of Set_X_To_Zero doesn't depend on any global variables. If you indicate this through a comment, as you often do in other languages, GNATprove can't verify that this is actually the case.

```plaintext
package Show_Depends_Contracts is

  type T is new Integer;

  procedure Swap (X, Y : in out T);
  -- The value of X (resp. Y) after the call depends only
  -- on the value of Y (resp. X) before the call

  X : Natural;
  procedure Set_X_To_Zero;
  -- The value of X after the call depends on no input

end Show_Depends_Contracts;
```

Like Global contracts, you specify a Depends contract in subprogram declarations using an aspect. Its value is a list of one or more dependency relations between the outputs and inputs of the subprogram. Each relation is represented as two lists of variable names separated by an arrow. On the left of each arrow are variables whose final value depends on the initial value of the variables you list on the right.

For example, here we indicate that the final value of each parameter of Swap depends only on the initial value of the other parameter. If the subprogram is a function, we list its result as an output, using the Result attribute, as we do for Get_Value_Of_X below.

Note: For more details on global contracts, see the SPARK User's Guide\(^\text{11}\).
package Show_Depends_Contracts is

  type T is new Integer;

  X, Y, Z : T := 0;

  procedure Swap (X, Y : in out T) with
    Depends => (X => Y,
                Y => X);
    -- X depends on the initial value of Y
    -- Y depends on the initial value of X

  function Get_Value_Of_X return T with
    Depends => (Get_Value_Of_X'Result => X);
    -- result depends on the initial value of X

  procedure Set_X_To_Y_Plus_Z with
    Depends => (X => (Y, Z));
    -- X depends on the initial values of Y and Z

  procedure Set_X_To_X_Plus_Y with
    Depends => (X => Y);
    -- X depends on Y and X's initial value

  procedure Do_Nothing (X : T) with
    Depends => (null => X);
    -- no output is affected by X

  procedure Set_X_To_Zero with
    Depends => (X => null);
    -- X depends on no input

end Show_Depends_Contracts;

Often, the final value of a variable depends on its own initial value. You can specify this in a concise way using the + character, as we did in the specification of Set_X_To_X_Plus_Y above. If there's more than one variable on the left of the arrow, a + means each variable depends on itself, not that they all depend on each other. You can write the corresponding dependency with (=>+) or without (=>+) whitespace.

If you have a program where an input isn't used to compute the final value of any output, you express that by writing null on the left of the dependency relation, as we did for the Do_Nothing subprogram above. You can only write one such dependency relation, which lists all unused inputs of the subprogram, and it must be written last. Such an annotation also silences flow analysis' warning about unused parameters. You can also write null on the right of a dependency relation to indicate that an output doesn't depend on any input. We do that above for the procedure Set_X_To_Zero.

2.3. Additional Verifications
2.4 Shortcomings

2.4.1 Modularity

Flow analysis is sound, meaning that if it doesn't output a message on some analyzed SPARK code, you can be assured that none of the errors it tests for can occur in that code. On the other hand, flow analysis often issues messages when there are, in fact, no errors. The first, and probably most common reason for this relates to modularity.

To scale flow analysis to large projects, verifications are usually done on a per-subprogram basis, including detection of uninitialized variables. To analyze this modularly, flow analysis needs to assume the initialization of inputs on subprogram entry and modification of outputs during subprogram execution. Therefore, each time a subprogram is called, flow analysis checks that global and parameter inputs are initialized and each time a subprogram returns, it checks that global and parameter outputs were modified.

This can produce error messages on perfectly correct subprograms. An example is `Set_X_To_Y_Plus_Z` below, which only sets its `out` parameter `X` when `Overflow` is `False`.

```ada
procedure Set_X_To_Y_Plus_Z
(Y, Z : Natural;
 X : out Natural;
 Overflow : out Boolean)
begin
 if Natural'Last - Z < Y then
  Overflow := True; -- X should be initialized on every path
 else
  Overflow := False;
  X := Y + Z;
 end if;
end Set_X_To_Y_Plus_Z;
```

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Set_X_To_Y_Plus_Z
MD5: be47cd769d2a7267c0bd1bb2ef0d6328

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
set_x_to_y_plus_z.adb:3:04: medium: "X" might not be initialized in "Set_X_To_Y_Plus_Z" [reason for check: OUT parameter should be initialized on return],
...[possible fix: initialize "X" on all paths or make "X" an IN OUT parameter]
gnatprove: unproved check messages considered as errors

The message means that flow analysis wasn't able to verify that the program didn't read an uninitialized variable. To solve this problem, you can either set `X` to a dummy value when there's an overflow or manually verify that `X` is never used after a call to `Set_X_To_Y_Plus_Z` that returned `True` as the value of `Overflow`.

2.4.2 Composite Types

Another common cause of false alarms is caused by the way flow analysis handles composite types. Let's start with arrays.

Flow analysis treats an entire array as single object instead of one object per element, so it considers modifying a single element to be a modification of the array as a whole. Obviously, this makes reasoning about which global variables are accessed less precise and hence the dependencies of those variables are also less precise. This also affects the ability to accurately detect reads of uninitialized data.

It's sometimes impossible for flow analysis to determine if an entire array object has been initialized. For example, after we write code to initialize every element of an unconstrained array $A$ in chunks, we may still receive a message from flow analysis claiming that the array isn't initialized. To resolve this issue, you can either use a simpler loop over the full range of the array, or (even better) an aggregate assignment, or, if that's not possible, verify initialization of the object manually.

Listing 10: show_composite_types_shortcoming.ads

```ada
package Show_Composite_Types_Shortcoming is
  type T is array (Natural range <>) of Integer;
  procedure Init_Chunks (A : out T);
  procedure Init_Loop (A : out T);
  procedure Init_Aggregate (A : out T);
end Show_Composite_Types_Shortcoming;
```

Listing 11: show_composite_types_shortcoming.adb

```ada
package body Show_Composite_Types_Shortcoming is
  procedure Init_Chunks (A : out T) is begin
    A (A'First) := 0;
    for I in A'First + 1 .. A'Last loop
      A (I) := 0;
    end loop;
    -- flow analysis doesn't know that A is initialized
  end Init_Chunks;

  procedure Init_Loop (A : out T) is begin
    for I in A'Range loop
      A (I) := 0;
    end loop;
    -- flow analysis knows that A is initialized
  end Init_Loop;

  procedure Init_Aggregate (A : out T) is begin
    A := (others => 0);
    -- flow analysis knows that A is initialized
  end Init_Aggregate;
end Show_Composite_Types_Shortcoming;
```
Flow analysis is more precise on record objects because it tracks the value of each component of a record separately within a single subprogram. So when a record object is initialized by successive assignments of its components, flow analysis knows that the entire object is initialized. However, record objects are still treated as single objects when analyzed as an input or output of a subprogram.

**Listing 12: show_record_flow_analysis.ads**

```ada
package Show_Record_Flow_Analysis is
  type Rec is record
    F1 : Natural;
    F2 : Natural;
  end record;
  procedure Init (R : out Rec);
end Show_Record_Flow_Analysis;
```

**Listing 13: show_record_flow_analysis.adb**

```ada
package body Show_Record_Flow_Analysis is
  procedure Init (R : out Rec) is
    begin
      R.F1 := 0;
      R.F2 := 0;
      -- R is initialized
    end Init;
end Show_Record_Flow_Analysis;
```

Flow analysis complains when a procedure call initializes only some components of a record object. It'll notify you of uninitialized components, as we see in subprogram Init_F2 below.
2.4.3 Value Dependency

Flow analysis is not value-dependent: it never reasons about the values of expressions, only whether they have been set to some value or not. As a consequence, if some execution path in a subprogram is impossible, but the impossibility can only be determined by looking at the values of expressions, flow analysis still considers that path feasible and may emit messages based on it believing that execution along such a path is possible.

For example, in the version of Absolute_Value below, flow analysis computes that R is uninitialized on a path that enters neither of the two conditional statements. Because it doesn't consider values of expressions, it can't know that such a path is impossible.
Listing 16: absolute_value.adb

```ada
procedure Absolute_Value
(X : Integer;
R : out Natural)
begin
if X < 0 then
  R := -X;
end if;
if X >= 0 then
  R := X;
end if;
-- flow analysis doesn't know that R is initialized
end Absolute_Value;
```

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Absolute_Value_1
MD5: 69c233d22afdfdac679bf379b353a8d4

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
absolute_value.adb:3:04: medium: "R" might not be initialized in "Absolute_Value"

[reason for check: OUT parameter should be initialized on return] [possible fix:
  initialize "R" on all paths or make "R" an IN OUT parameter]
gnatprove: unproved check messages considered as errors

To avoid this problem, you should make the control flow explicit, as in this second version of Absolute_Value:

Listing 17: absolute_value.adb

```ada
procedure Absolute_Value
(X : Integer;
R : out Natural)
begin
if X < 0 then
  R := -X;
else
  R := X;
end if;
-- flow analysis knows that R is initialized
end Absolute_Value;
```

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Absolute_Value_2
MD5: 9c773547f81e02a7aa1b45132b105937

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
2.4.4 Contract Computation

The final cause of unexpected flow messages that we'll discuss also comes from inaccuracy in computations of contracts. As we explained earlier, both Global and Depends contracts are optional, but GNATprove uses their data for some of its analysis.

For example, flow analysis can't detect reads from uninitialized variables without knowing the set of variables accessed. It needs to analyze and check both the Depends contracts you wrote for a subprogram and those you wrote for callers of that subprogram. Since each flow contract on a subprogram depends on the flow contracts of all the subprograms called inside its body, this computation can often be quite time-consuming. Therefore, flow analysis sometimes trades-off the precision of this computation against the time a more precise computation would take.

This is the case for Depends contracts, where flow analysis simply assumes the worst, that each subprogram's output depends on all of that subprogram's inputs. To avoid this assumption, all you have to do is supply contracts when default ones are not precise enough. You may also want to supply Global contracts to further speed up flow analysis on larger programs.

2.5 Code Examples / Pitfalls

2.5.1 Example #1

The procedure Search_Array searches for an occurrence of element E in an array A. If it finds one, it stores the index of the element in Result. Otherwise, it sets Found to False.

Listing 18: show_search_array.ads

```ada
package Show_Search_Array is
  type Array_Of_Positives is array (Natural range <>) of Positive;

  procedure Search_Array
    (A    : Array_Of_Positives;
     E    : Positive;
     Result : out Integer;
     Found  : out Boolean);

end Show_Search_Array;
```

Listing 19: show_search_array.adb

```ada
package body Show_Search_Array is

  procedure Search_Array
    (A    : Array_Of_Positives;
     E    : Positive;
     Result : out Integer;
     Found  : out Boolean) is
  begin
    for I in A'Range loop
      if A (I) = E then
        Result := I;
        Found := True;
        return;
      end if;
    end loop;

end Search_Array;
```

(continues on next page)
16. Found := False;
   end Search_Array;
end Show_Search_Array;

**Code block metadata**

Project: Courses.Intro_To_Spark.Flow Analysis.Example_01
MD5: d2a27a5bde247767e2f6cd2d42a2d629

**Prover output**

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_search_array.ads:8:07: medium: "Result" might not be initialized in "Search_Array" [reason for check: OUT parameter should be initialized on return].
   [possible fix: initialize "Result" on all paths or make "Result" an IN OUT parameter]
gnatprove: unproved check messages considered as errors

GNATprove produces a message saying that Result is possibly uninitialized on return. There are perfectly legal uses of the function Search_Array, but flow analysis detects that Result is not initialized on the path that falls through from the loop. Even though this program is correct, you shouldn't ignore the message: it means flow analysis cannot guarantee that Result is always initialized at the call site and so assumes any read of Result at the call site will read initialized data. Therefore, you should either initialize Result when Found is false, which silences flow analysis, or verify this assumption at each call site by other means.

### 2.5.2 Example #2

To avoid the message previously issued by GNATprove, we modify Search_Array to raise an exception when E isn't found in A:

**Listing 20: show_search_array.ads**

```ada
package Show_Search_Array is
  type Array_Of_Positives is array (Natural range <>) of Positive;
  Not_Found : exception;

  procedure Search_Array
    (A : Array_Of_Positives;
     E : Positive;
     Result : out Integer);
end Show_Search_Array;
```

**Listing 21: show_search_array.adb**

```ada
package body Show_Search_Array is
  procedure Search_Array
    (A : Array_Of_Positives;
     E : Positive;
     Result : out Integer)
  begin
    for I in A'Range loop
```

(continues on next page)
if A(I) = E then
    Result := I;
    return;
end if;
end loop;
raise Not_Found;
end Search_Array;
end Show_Search_Array;

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Example_02
MD5: fa159faeb68974b1af3de2112e086b16

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_search_array.adb:14:07: medium: exception might be raised
gnatprove: unproved check messages considered as errors

Flow analysis doesn't emit any messages in this case, meaning it can verify that Result can't be read in SPARK code while uninitialized. But why is that, since Result is still not initialized when E is not in A? This is because the exception, Not_Found, can never be caught within SPARK code (SPAK doesn't allow exception handlers). However, the GNATprove tool also tries to ensure the absence of runtime errors in SPARK code, so tries to prove that Not_Found is never raised. When it can't do that here, it produces a different message.

2.5.3 Example #3

In this example, we're using a discriminated record for the result of Search_Array instead of conditionally raising an exception. By using such a structure, the place to store the index at which E was found exists only when E was indeed found. So if it wasn't found, there's nothing to be initialized.

Listing 22: show_search_array.ads

package Show_Search_Array is
    type Array_Of_Positives is array (Natural range <>) of Positive;
    type Search_Result (Found : Boolean := False) is record
        case Found is
            when True =>
                Content : Integer;
            when False => null;
        end case;
    end record;

    procedure Search_Array
    (A : Array_Of_Positives;
    E : Positive;
    Result : out Search_Result)
    with Pre => not Result'Constrained;
end Show_Search_Array;
Listing 23: show_search_array.adb

```ada
package body Show_Search_Array is

  procedure Search_Array
    (A : Array_Of_Positives;
     E : Positive;
     Result : out Search_Result) is
  begin
    for I in A'Range loop
      if A (I) = E then
        Result := (Found => True,
                   Content => I);
        return;
      end if;
    end loop;
    Result := (Found => False);
  end Search_Array;

end Show_Search_Array;
```

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Example_03
MD5: 1d5ec5d78185fd75499b90b3d21f6ae2

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_search_array.adb:10:20: info: discriminant check proved
show_search_array.adb:15:14: info: discriminant check proved
show_search_array.ads:16:07: info: initialization of "Result" proved

This example is correct and flow analysis doesn't issue any message: it can verify both that no uninitialized variables are read in Search_Array's body, and that all its outputs are set on return. We've used the attribute Constrained in the precondition of Search_Array to indicate that the value of the Result in argument can be set to any variant of the record type Search_Result, specifically to either the variant where E was found and where it wasn't.

2.5.4 Example #4

The function Size_Of_Biggest_Increasing_Sequence is supposed to find all sequences within its parameter A that contain elements with increasing values and returns the length of the longest one. To do this, it calls a nested procedure Test_Index iteratively on all the elements of A. Test_Index checks if the sequence is still increasing. If so, it updates the largest value seen so far in this sequence. If not, it means it's found the end of a sequence, so it computes the size of that sequence and stores it in Size_Of_Seq.

Listing 24: show_biggest_increasing_sequence.ads

```ada
package Show_Biggest_Increasing_Sequence is

  type Array_Of_Positives is array (Integer range <>) of Positive;

  function Size_Of_Biggest_Increasing_Sequence (A : Array_Of_Positives)
    return Natural;

end Show_Biggest_Increasing_Sequence;
```

(continues on next page)
Introduction to SPARK

(continued from previous page)

Listing 25: show_biggest_increasing_sequence.adb

```ada
package body Show_Biggest_Increasing_Sequence is
  function Size_Of_Biggest_Increasing_Sequence (A : Array_Of_Positives)
    return Natural
  is
    Max : Natural;
    End_Of_Seq : Boolean;
    Size_Of_Seq : Natural;
    Beginning : Integer;

    procedure Test_Index (Current_Index : Integer) is
    begin
      if A (Current_Index) > Max then
        Max := A (Current_Index);
        End_Of_Seq := False;
      else
        Max := 0;
        End_Of_Seq := True;
        Size_Of_Seq := Current_Index - Beginning;
        Beginning := Current_Index;
      end if;
    end Test_Index;

    Biggest_Seq : Natural := 0;

    begin
      for I in A’Range loop
        Test_Index (I);
        if End_Of_Seq then
          Biggest_Seq := Natural’Max (Size_Of_Seq, Biggest_Seq);
        end if;
      end loop;
      return Biggest_Seq;
    end Show_Biggest_Increasing_Sequence;

end Show_Biggest_Increasing_Sequence;
```

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Example_04
MD5: e6083665827d9dee4e0bdce4c1e962f

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_biggest_increasing_sequence.adb:13:34: medium: "Max" might not be initialized,
in call inlined at show_biggest_increasing_sequence.adb:28
show_biggest_increasing_sequence.adb:19:44: medium: "Beginning" might not be initialized,
in call inlined at show_biggest_increasing_sequence.adb:28
show_biggest_increasing_sequence.adb:30:41: medium: "Size_Of_Seq" might not be initialized

gnatprove: unproved check messages considered as errors

However, this example is not correct. Flow analysis emits messages for Test_Index stating that Max, Beginning, and Size_Of_Seq should be initialized before being read. Indeed,
Introduction to SPARC

when you look carefully, you see that both Max and Beginning are missing initializations because they are read in Test_Index before being written. As for Size_Of_Seq, we only read its value when End_Of_Seq is true, so it actually can't be read before being written, but flow analysis isn't able to verify its initialization by using just flow information.

The call to Test_Index is automatically inlined by GNATprove, which leads to another messages above. If GNATprove couldn't inline the call to Test_Index, for example if it was defined in another unit, the same messages would be issued on the call to Test_Index.

2.5.5 Example #5

In the following example, we model permutations as arrays where the element at index I is the position of the I'th element in the permutation. The procedure Init initializes a permutation to the identity, where the I'th elements is at the I'th position. Cyclic_Permutation calls Init and then swaps elements to construct a cyclic permutation.

Listing 26: show_permutation.ads

```
package Show_Permutation is
  type Permutation is array (Positive range <>) of Positive;
  procedure Swap (A : in out Permutation;
                   I, J : Positive);
  procedure Init (A : out Permutation);
  function Cyclic_Permutation (N : Natural) return Permutation;
end Show_Permutation;
```

Listing 27: show_permutation.adb

```
package body Show_Permutation is
  procedure Swap (A : in out Permutation;
                  I, J : Positive)
     is
    begin
      Tmp : Positive := A (I);
      A (I) := A (J);
      A (J) := Tmp;
      Swap;
    end Swap;
  procedure Init (A : out Permutation) is
    begin
      A (A'First) := A'First;
      for I in A'First .. A'Last loop
        A (I) := I;
      end loop;
    end Init;
  function Cyclic_Permutation (N : Natural) return Permutation is
    begin
      Init (A);
      for I in A'First .. A'Last - 1 loop
        Swap (A, I, I + 1);
      end loop;
    return A;
```

(continues on next page)
### Code block metadata

**Project:** Courses.Intro_To_Spark.Flow_Analysis.Example_05  
**MD5:** 219b06617c636c10543128d77f90fcee

#### Prover output

Phase 1 of 2: generation of Global contracts ...  
Phase 2 of 2: analysis of data and information flow ...  
show_permutation.ads:8:20: medium: "A" might not be initialized in "Init" [reason,  
→ for check: OUT parameter should be fully initialized on return] [possible fix:  
→ initialize "A" on all paths, make "A" an IN OUT parameter or annotate it with,  
→ aspect Relaxed Initialization]  
gnatprove: unproved check messages considered as errors

This program is correct. However, flow analysis will nevertheless still emit messages because it can't verify that every element of A is initialized by the loop in Init. This message is a false alarm. You can either ignore it or justify it safely.

### 2.5.6 Example #6

This program is the same as the previous one except that we've changed the mode of A in the specification of Init to **in out** to avoid the message from flow analysis on array assignment.

---

**Listing 28:** show_permutation.ads

```ada
package Show_Permutation is
  type Permutation is array (Positive range <>) of Positive;
  procedure Swap (A : in out Permutation;  
                  I, J : Positive);
  procedure Init (A : in out Permutation);
  function Cyclic_Permutation (N : Natural) return Permutation;
end Show_Permutation;
```

**Listing 29:** show_permutation.adb

```ada
package body Show_Permutation is
  procedure Swap (A : in out Permutation;  
                  I, J : Positive)
  is
    Tmp : Positive := A (I);
  begin
    A (I) := A (J);
    A (J) := Tmp;
  end Swap;
  procedure Init (A : in out Permutation) is
```

(continues on next page)
begin  
  A (A'First) := A'First;  
  for I in A'First + 1 .. A'Last loop  
    A (I) := I;  
    end loop;  
  end Init;  
  
  function Cyclic_Permutation (N : Natural) return Permutation is  
    A : Permutation (1 .. N);  
    begin  
      Init (A);  
      for I in A'First .. A'Last - 1 loop  
        Swap (A, I, I + 1);  
        end loop;  
      return A;  
    end Cyclic_Permutation;  
  end Show_Permutation;

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Example_06
MD5: 61406d9a66dda71630c74c12f3d67936

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_permutation.adb:23:13: high: "A" is not initialized
gnatprove: unproved check messages considered as errors

This program is not correct. Changing the mode of a parameter that should really be out to in out to silence a false alarm is not a good idea. Not only does this obfuscate the specification of Init, but flow analysis emits a message on the procedure where A is not initialized, as shown by the message in Cyclic_Permutation.

2.5.7 Example #7

Incr_Step_Function takes an array A as an argument and iterates through A to increment every element by the value of Increment, saturating at a specified threshold value. We specified a Global contract for Incr_Until_Threshold.

Listing 30: show_increments.ads

package Show_Increments is  
  type Array_Of_Positives is array (Natural range <>) of Positive;  
  Increment : constant Natural := 10;  
  procedure Incr_Step_Function (A : in out Array_Of_Positives);  
end Show_Increments;

Listing 31: show_increments.adb

package body Show_Increments is

(continues on next page)
procedure Incr_Step_Function (A : in out Array_Of_Positives) is

    Threshold : Positive := Positive'Last;

procedure Incr_Until_Threshold (I : Integer) with
    Global => (Input => Threshold,
              In_Out => A);

procedure Incr_Until_Threshold (I : Integer) is
begin
    if Threshold - Increment <= A (I) then
        A (I) := Threshold;
    else
        A (I) := A (I) + Increment;
    end if;
end Incr_Until_Threshold;

begin
    for I in A'Range loop
        if I > A'First then
            Threshold := A (I - 1);
        end if;
        Incr_Until_Threshold (I);
    end loop;
end Incr_Step_Function;

end Show_Increments;

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Example_07
MD5: 8e28a005cd9d78947e4bfc60db708bf5

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_increments.adb:8:09: info: data dependencies proved

Everything is fine here. Specifically, the Global contract is correct. It mentions both Threshold, which is read but not written in the procedure, and A, which is both read and written. The fact that A is a parameter of an enclosing unit doesn't prevent us from using it inside the Global contract; it really is global to Incr_Until_Threshold. We didn't mention Increment since it's a static constant.

2.5.8 Example #8

We now go back to the procedure Test_Index from Example #4 (page 36) and correct the missing initializations. We want to know if the Global contract of Test_Index is correct.

Listing 32: show_biggest_increasing_sequence.ads

package Show_Biggest_Increasing_Sequence is

    type Array_Of_Positives is array (Integer range <>) of Positive;

function Size_Of_Biggest_Increasing_Sequence (A : Array_Of_Positives)
    return Natural;

(continues on next page)
Introduction to SPARK

(continued from previous page)

end Show_Biggest_Increasing_Sequence;

Listing 33: show_biggest_increasing_sequence.adb

package body Show_Biggest_Increasing_Sequence is

function Size_Of_Biggest_Increasing_Sequence (A : Array_Of_Positives)
  return Natural
is
  Max : Natural := 0;
  End_Of_Seq : Boolean;
  Size_Of_Seq : Natural := 0;
  Beginning : Integer := A'First - 1;

procedure Test_Index (Current_Index : Integer) with
  Global => (In_Out => (Beginning, Max, Size_Of_Seq),
              Output => End_Of_Seq,
              Input => Current_Index)
is
  begin
  if A (Current_Index) <= Max then
    Max := A (Current_Index);
    End_Of_Seq := False;
  else
    Max := 0;
    End_Of_Seq := True;
    Size_Of_Seq := Current_Index - Beginning;
    Beginning := Current_Index;
  end if;
end Test_Index;

Biggest_Seq : Natural := 0;

begin
  for I in A'Range loop
    Test_Index (I);
    if End_Of_Seq then
      Biggest_Seq := Natural'Max (Size_Of_Seq, Biggest_Seq);
    end if;
  end loop;
  return Biggest_Seq;
end Size_Of_Biggest_Increasing_Sequence;

end Show_Biggest_Increasing_Sequence;

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Example_08
MD5: 86fb934c32a38f6841ef736780b2e3b2

Prover output

Phase 1 of 2: generation of Global contracts ...
show_biggest_increasing_sequence.adb:14:30: error: global item cannot reference parameter of subprogram "Test Index"
gnatprove: error during generation of Global contracts

The contract in this example is not correct: Current_Index is a parameter of Test_Index, so we shouldn't reference it as a global variable. Also, we should have listed variable A from the outer scope as an Input in the Global contract.
2.5.9 Example #9

Next, we change the Global contract of Test_Index into a Depends contract. In general, we don't need both contracts because the set of global variables accessed can be deduced from the Depends contract.

Listing 34: show_biggest_increasing_sequence.ads

```ada
package Show_Biggest_Increasing_Sequence is

  type Array_Of_Positives is array (Integer range <>) of Positive;

  function Size_Of_Biggest_Increasing_Sequence (A : Array_Of_Positives) return Natural;

end Show_Biggest_Increasing_Sequence;
```

Listing 35: show_biggest_increasing_sequence.adb

```ada
package body Show_Biggest_Increasing_Sequence is

  function Size_Of_Biggest_Increasing_Sequence (A : Array_Of_Positives) return Natural is
    Max : Natural := 0;
    End_Of_Seq : Boolean;
    Size_Of_Seq : Natural := 0;
    Beginning : Integer := A'First - 1;

    procedure Test_Index (Current_Index : Integer) with Depends => ((Max, End_Of_Seq) => (A, Current_Index, Max),
    (Size_Of_Seq, Beginning) => + (A, Current_Index, Max, Beginning))
    is
      begin
        if A (Current_Index) >= Max then
          Max := A (Current_Index);
          End_Of_Seq := False;
        else
          Max := 0;
          End_Of_Seq := True;
          Size_Of_Seq := Current_Index - Beginning;
          Beginning := Current_Index;
        end if;

      end Test_Index;

      Biggest_Seq : Natural := 0;

      begin
        for I in A'Range loop
          Test_Index (I);
          if End_Of_Seq then
            Biggest_Seq := Natural'Max (Size_Of_Seq, Biggest_Seq);
          end if;
        end loop;
      end Size_Of_Biggest_Increasing_Sequence;

end Show_Biggest_Increasing_Sequence;
```
Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_biggest_increasing_sequence.adb:7:07: info: initialization of "End_Of_Seq"
 proved
show_biggest_increasing_sequence.adb:11:17: info: initialization of "End_Of_Seq"
 proved
show_biggest_increasing_sequence.adb:12:09: info: flow dependencies proved

This example is correct. Some of the dependencies, such as Size_Of_Seq depending on Beginning, come directly from the assignments in the subprogram. Since the control flow influences the final value of all of the outputs, the variables that are being read, A, Current_Index, and Max, are present in every dependency relation. Finally, the dependencies of Size_Of_Eq and Beginning on themselves are because they may not be modified by the subprogram execution.

2.5.10 Example #10

The subprogram Identity swaps the value of its parameter two times. Its Depends contract says that the final value of X only depends on its initial value and likewise for Y.

Listing 36: show_swap.ads

```ada
package Show_Swap is

   procedure Swap (X, Y : in out Positive);

   procedure Identity (X, Y : in out Positive) with
      Depends => (X => X,
                  Y => Y);

end Show_Swap;
```

Listing 37: show_swap.adb

```ada
package body Show_Swap is

   procedure Swap (X, Y : in out Positive) is
      Tmp : constant Positive := X;
   begin
      X := Y;
      Y := Tmp;
   end Swap;

   procedure Identity (X, Y : in out Positive) is
   begin
      Swap (X, Y);
      Swap (Y, X);
   end Identity;

end Show_Swap;
```

Code block metadata

Project: Courses.Intro_To_Spark.Flow_Analysis.Example_10
MD5: 8567e3e1e5bbc190f62dd483785d078a
Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
show_swap.ads:6:18: medium: missing dependency "X => Y"
show_swap.ads:7:18: medium: missing dependency "Y => X"
gnatprove: unproved check messages considered as errors

This code is correct, but flow analysis can't verify the Depends contract of Identity because we didn't supply a Depends contract for Swap. Therefore, flow analysis assumes that all outputs of Swap, X and Y, depend on all its inputs, both X and Y's initial values. To prevent this, we should manually specify a Depends contract for Swap.
CHAPTER THREE

PROOF OF PROGRAM INTEGRITY

This section presents the proof capability of GNATprove, a major tool for the SPARK language. We focus here on the simpler proofs that you’ll need to write to verify your program's integrity. The primary objective of performing proof of your program's integrity is to ensure the absence of runtime errors during its execution.

The analysis steps discussed here are only sound if you've previously performed Flow Analysis (page 21). You shouldn't proceed further if you still have unjustified flow analysis messages for your program.

3.1 Runtime Errors

There’s always the potential for errors that aren't detected during compilation to occur during a program's execution. These errors, called runtime errors, are those targeted by GNATprove.

There are various kinds of runtime errors, the most common being references that are out of the range of an array (buffer overflow in Ada), subtype range violations, overflows in computations, and divisions by zero. The code below illustrates many examples of possible runtime errors, all within a single statement. Look at the assignment statement setting the I + J’th cell of an array A to the value P /Q.

Listing 1: show_runtime_errors.ads

```ada
package Show_Runtime_Errors is
    type Nat_Array is array (Integer range <>) of Natural;
    procedure Update (A : in out Nat_Array; I, J, P, Q : Integer);
end Show_Runtime_Errors;
```

Listing 2: show_runtime_errors.adb

```ada
package body Show_Runtime_Errors is
    procedure Update (A : in out Nat_Array; I, J, P, Q : Integer) is
        begin
            A (I + J) := P / Q;
        end Update;
end Show_Runtime_Errors;
```

Code block metadata

There are quite a number of errors that may occur when executing this code. If we don't know anything about the values of I, J, P, and Q, we can't rule out any of those errors.

First, the computation of I + J can overflow, for example if I is \texttt{Integer'}Last and J is positive.

\begin{verbatim}
A (\texttt{Integer'}Last + 1) := P / Q;
\end{verbatim}

Next, the sum, which is used as an array index, may not be in the range of the index of the array.

\begin{verbatim}
A (A'Last + 1) := P / Q;
\end{verbatim}

On the other side of the assignment, the division may also overflow, though only in the very special case where P is \texttt{Integer'}First and Q is -1 because of the asymmetric range of signed integer types.

\begin{verbatim}
A (I + J) := \texttt{Integer'}First / -1;
\end{verbatim}

The division is also not allowed if Q is 0.

\begin{verbatim}
A (I + J) := P / 0;
\end{verbatim}

Finally, since the array contains natural numbers, it's also an error to store a negative value in it.

\begin{verbatim}
A (I + J) := 1 / -1;
\end{verbatim}

The compiler generates checks in the executable code corresponding to each of those runtime errors. Each check raises an exception if it fails. For the above assignment statement, we can see examples of exceptions raised due to failed checks for each of the different cases above.
These runtime checks are costly, both in terms of program size and execution time. It may be appropriate to remove them if we can statically ensure they aren't needed at runtime, in other words if we can prove that the condition tested for can never occur.

This is where the analysis done by GNATprove comes in. It can be used to demonstrate statically that none of these errors can ever occur at runtime. Specifically, GNATprove logically interprets the meaning of every instruction in the program. Using this interpretation, GNATprove generates a logical formula called a verification condition for each check that would otherwise be required by the Ada (and hence SPARK) language.

GNATprove then passes these verification conditions to an automatic prover, stated as conditions that must be true to avoid the error. If every such condition can be validated by a prover (meaning that it can be mathematically shown to always be true), we've been able to prove that no error can ever be raised at runtime when executing that program.

3.2 Modularity

To scale to large programs, GNATprove performs proofs on a per-subprogram basis by relying on preconditions and postconditions to properly summarize the input and output state of each subprogram. More precisely, when verifying the body of a subprogram, GNATprove assumes it knows nothing about the possible initial values of its parameters and of the global variables it accesses except what you state in the subprogram's precondition. If you don't specify a precondition, it can't make any assumptions.

For example, the following code shows that the body of Increment can be successfully verified: its precondition constrains the value of its parameter X to be less than Integer'Last so we know the overflow check is always false.
**Introduction to SPARK**

In the same way, when a subprogram is called, GNATprove assumes its `out` and `in out` parameters and the global variables it writes can be modified in any way compatible with their postconditions. For example, since `Increment` has no postcondition, GNATprove doesn't know that the value of `X` after the call is always less than `Integer'Last`. Therefore, it can't prove that the addition following the call to `Increment` can't overflow.

**Listing 3: show_modularity.adb**

```adbsmall"
procedure Show_Modularity is
    procedure Increment (X : in out Integer) with
        Pre => X < Integer'Last is
    begin
        X := X + 1;
        -- info: overflow check proved
    end Increment;
    X : Integer;
begin
    X := Integer'Last - 2;
    Increment (X);
    -- After the call, GNATprove no longer knows the value of X
    X := X + 1;
    -- medium: overflow check might fail
end Show_Modularity;
```

**Code block metadata**

Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Modularity_1
MD5: ca8ff8d29792fd5a06f7cb0158e13609

**Prover output**

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_modularity.adb:6:14: info: overflow check proved
show_modularity.adb:10:04: info: initialization of "X" proved
show_modularity.adb:13:04: info: precondition proved
show_modularity.adb:16:11: medium: overflow check might fail, cannot prove upper bound for `X + 1` [reason for check: result of addition must fit in a 32-bits machine integer] [possible fix: call at line 13 should mention `X` (for argument `X`) in a postcondition]
gnatprove: unproved check messages considered as errors

### 3.2.1 Exceptions

There are two cases where GNATprove doesn't require modularity and hence doesn't make the above assumptions. First, local subprograms without contracts can be inlined if they're simple enough and are neither recursive nor have multiple return points. If we remove the contract from `Increment`, it fits the criteria for inlining.

**Listing 4: show_modularity.adb**

```adbsmall"
procedure Show_Modularity is
    procedure Increment (X : in out Integer) is
    begin
        X := X + 1;
        -- info: overflow check proved, in call inlined at...
```

(continues on next page)
Introduction to SPARK

(continued from previous page)

```ada
end Increment;

X : Integer;

begin
  X := Integer'Last - 2;
  Increment (X);
  X := X + 1;
  -- info: overflow check proved
end Show_Modularity;
```

**Code block metadata**

Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Modularity_2
MD5: 448d576897c3e4606cd4b90621aad63a

**Prover output**

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_modularity.adb:5:14: info: overflow check proved, in call inlined at show_modularity.adb:12
show_modularity.adb:9:04: info: initialization of "X" proved
show_modularity.adb:13:11: info: overflow check proved

GNATprove now sees the call to Increment exactly as if the increment on X was done outside that call, so it can successfully verify that neither addition can overflow.

**Note:** For more details on contextual analysis of subprograms, see the SPARK User's Guide\(^{14}\).

The other case involves functions. If we define a function as an expression function, with or without contracts, GNATprove uses the expression itself as the postcondition on the result of the function.

In our example, replacing Increment with an expression function allows GNATprove to successfully verify the overflow check in the addition.

**Listing 5: show_modularity.adb**

```ada
procedure Show_Modularity is

  function Increment (X : Integer) return Integer is
    (X + 1)
    -- info: overflow check proved
    with Pre => X < Integer'Last;

  X : Integer;

begin
  X := Integer'Last - 2;
  X := Increment (X);
  X := X + 1;
  -- info: overflow check proved
end Show_Modularity;
```

**Code block metadata**


3.2. Modularity
### 3.3 Contracts

Ada contracts are perfectly suited for formal verification, but are primarily designed to be checked at runtime. When you specify the `-gnata` switch, the compiler generates code that verifies the contracts at runtime. If an Ada contract isn't satisfied for a given subprogram call, the program raises the `Assert_Failure` exception. This switch is particularly useful during development and testing, but you may also retain run-time execution of assertions, and specifically preconditions, during the program's deployment to avoid an inconsistent state.

Consider the incorrect call to `Increment` below, which violates its precondition. One way to detect this error is by compiling the function with assertions enabled and testing it with inputs that trigger the violation. Another way, one that doesn't require guessing the needed inputs, is to run GNATprove.

Listing 6: `show_precondition_violation.adb`

```ada
procedure Show_Precondition_Violation is

  procedure Increment (X : in out Integer) with
    Pre => X < Integer’Last is
  begin
    X := X + 1;
  end Increment;

  X : Integer;

begin
  X := Integer’Last;
  Increment (X);
end Show_Precondition_Violation;
```

---

**Note:** For more details on expression functions, see the SPARK User's Guide.

---

Introduction to SPARK

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_preconditionViolation.adb:13:04: medium: precondition might fail
gnatprove: unproved check messages considered as errors

Runtime output

raised ADAASSERTIONSASSERTION_ERROR: failed precondition from show_precondition_ violation.adb:4

Similarly, consider the incorrect implementation of function Absolute below, which violates its postcondition. Likewise, one way to detect this error is by compiling the function with assertions enabled and testing with inputs that trigger the violation. Another way, one which again doesn't require finding the inputs needed to demonstrate the error, is to run GNATprove.

Listing 7: show_postconditionViolation.adb

```ada
procedure Show_Postcondition_Violation
is
    procedure Absolute (X : in out Integer) with
    Post => X > 0
begin
    if X > 0 then
        X := -X;
    end if;
    end Absolute;

begin
    X := 1;
    Absolute (X);
end Show_Postcondition_Violation;
```

The benefits of dynamically checking contracts extends beyond making testing easier. Early failure detection also allows an easier recovery and facilitates debugging, so you may want to enable these checks at runtime to terminate execution before some damaging or hard-to-debug action occurs.

GNATprove statically analyses preconditions and postconditions. It verifies preconditions every time a subprogram is called, which is the runtime semantics of contracts. Postconditions, on the other hand, are verified once as part of the verification of the subprogram's
body. For example, GNATprove must wait until Increment is improperly called to detect the precondition violation, since a precondition is really a contract for the caller. On the other hand, it doesn't need Absolute to be called to detect that its postcondition doesn't hold for all its possible inputs.

**Note:** For more details on pre and postconditions, see the SPARK User's Guide\(^\text{16}\).

### 3.3.1 Executable Semantics

Expressions in Ada contracts have the same semantics as Boolean expressions elsewhere, so runtime errors can occur during their computation. To simplify both debugging of assertions and combining testing and static verification, the same semantics are used by GNATprove.

While proving programs, GNATprove verifies that no error can ever be raised during the execution of the contracts. However, you may sometimes find those semantics too heavy, in particular with respect to overflow checks, because they can make it harder to specify an appropriate precondition. We see this in the function Add below.

Listing 8: show_executable_semantics.adb

```
procedure Show_Executable_Semantics
  with SPARK_Mode => On
is
  function Add (X, Y : Integer) return Integer is (X + Y)
    with Pre => X + Y in Integer;

  X : Integer;
begin
  X := Add (Integer'Last, 1);
end Show_Executable_Semantics;
```

**Build output**

show_executable_semantics.adb:5:24: warning: explicit membership test may be optimized away [enabled by default]
show_executable_semantics.adb:5:24: warning: use 'Valid attribute instead [enabled, by default]

**Prover output**

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_executable_semantics.adb:5:20: medium: overflow check might fail, cannot prove lower bound for X + Y [reason for check: result of addition must fit in a 32-bits machine integer] [possible fix: use pragma Overflow_Mode or switch gnato13 or unit Ada.Numerics.Big_Numbers.Big_Integers]
show_executable_semantics.adb:9:09: medium: precondition might fail, cannot prove upper bound for Add (Integer'Last, 1)
gnatprove: unproved check messages considered as errors

**Runtime output**

GNATprove issues a message on this code warning about a possible overflow when computing the sum of \( X \) and \( Y \) in the precondition. Indeed, since expressions in assertions have normal Ada semantics, this addition can overflow, as you can easily see by compiling and running the code that calls `Add` with arguments `Integer'Last` and 1.

On the other hand, you sometimes may prefer GNATprove to use the mathematical semantics of addition in contracts while the generated code still properly verifies that no error is ever raised at runtime in the body of the program. You can get this behavior by using the compiler switch `-gnato??` (for example `-gnato13`), which allows you to independently set the overflow mode in code (the first digit) and assertions (the second digit). For both, you can either reduce the number of overflow checks (the value 2), completely eliminate them (the value 3), or preserve the default Ada semantics (the value 1).

Note: For more details on overflow modes, see the SPARK User's Guide\(^\text{17}\).

### 3.3.2 Additional Assertions and Contracts

As we've seen, a key feature of SPARK is that it allows us to state properties to check using assertions and contracts. SPARK supports preconditions and postconditions as well as assertions introduced by the `Assert` pragma.

The SPARK language also includes new contract types used to assist formal verification. The new pragma `Assume` is treated as an assertion during execution but introduces an assumption when proving programs. Its value is a Boolean expression which GNATprove assumes to be true without any attempt to verify that it's true. You'll find this feature useful, but you must use it with great care. Here's an example of using it.

```
procedure Incr (X : in out Integer) is
begin
  pragma Assume (X < Integer'Last);
  X := X + 1;
end Incr;
```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Pragma_Assume
MD5: bfbc4b8aca259d7516b6acaee571f8c2

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
incr.adb:4:11: info: overflow check proved

Note: For more details on pragma `Assume`, see the SPARK User's Guide\(^\text{18}\).

The `Contract_Cases` aspect is another construct introduced for GNATprove, but which also acts as an assertion during execution. It allows you to specify the behavior of a subprogram


using a disjunction of cases. Each element of a Contract_Cases aspect is a guard, which is evaluated before the call and may only reference the subprogram's inputs, and a consequence. At each call of the subprogram, one and only one guard is permitted to evaluate to True. The consequence of that case is a contract that's required to be satisfied when the subprogram returns.

Listing 10: absolute.adb

```ada
procedure Absolute (X : in out Integer) with
  Pre => X > Integer'First,
  Contract_Cases => (X < 0 => X = -X'Old,
   X >= 0 => X = X'Old)
begin
  if X < 0 then
    X := -X;
  end if;
end Absolute;
```

Similarly to how it analyzes a subprogram's precondition, GNATprove verifies the Contract_Cases only once. It verifies the validity of each consequence (given the truth of its guard) and the disjointness and completeness of the guard conditions (meaning that exactly one guard must be true for each possible set of input values).

Note: For more details on Contract_Cases, see the SPARK User's Guide.  

3.4 Debugging Failed Proof Attempts

GNATprove may report an error while verifying a program for any of the following reasons:

- there might be an error in the program; or
- the property may not be provable as written because more information is required; or
- the prover used by GNATprove may be unable to prove a perfectly valid property.

We spend the remainder of this section discussing the sometimes tricky task of debugging failed proof attempts.
3.4.1 Debugging Errors in Code or Specification

First, let's discuss the case where there's indeed an error in the program. There are two possibilities: the code may be incorrect or, equally likely, the specification may be incorrect. As an example, there's an error in our procedure Incr_Until below which makes its Contract_Cases unprovable.

Listing 11: show_failed_proof_attempt.ads

```ada
package Show_Failed_Proof_Attempt is

  Incremented : Boolean := False;

  procedure Incr_Until (X : in out Natural) with
  Contract_Cases =>
    (Incremented => X > X'Old,
     others => X = X'Old);

end Show_Failed_Proof_Attempt;
```

Listing 12: show_failed_proof_attempt.adb

```ada
package body Show_Failed_Proof_Attempt is

  procedure Incr_Until (X : in out Natural) is
  begin
    if X < 1000 then
      X := X + 1;
      Incremented := True;
    else
      Incremented := False;
    end if;
  end Incr_Until;

end Show_Failed_Proof_Attempt;
```

Since this is an assertion that can be executed, it may help you find the problem if you run the program with assertions enabled on representative sets of inputs. This allows you to find bugs in both the code and its contracts. In this case, testing Incr_Until with an input greater than 1000 raises an exception at runtime.

Listing 13: show_failed_proof_attempt.ads

```ada
package Show_Failed_Proof_Attempt is

  Incremented : Boolean := False;

  procedure Incr_Until (X : in out Natural) with
  (continues on next page)
```

3.4. Debugging Failed Proof Attempts
Introduction to SPARK

(continued from previous page)

```ada
contract

global

end show_failed_proof_attempt;

Listing 14: show_failed_proof_attempt.adb

```ada
package body Show_Failed_Proof_Attempt is

procedure Incr_Until (X : in out Natural) is

begin

if X < 1000 then

X := X + 1;

Incremented := True;

else

Incremented := False;

end if;

end Incr_Until;

end Show_Failed_Proof_Attempt;

Listing 15: main.adb

```ada
with Show_Failed_Proof_Attempt; use Show_Failed_Proof_Attempt;

procedure Main is

Dummy : Integer;

begin

Dummy := 0;

Incr_Until (Dummy);

Dummy := 1000;

Incr_Until (Dummy);

end Main;

```ada

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Failed_Proof_Attempt_2
MD5: bd87cb0f64a6468eaab3cad1678271db

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_failed_proof_attempt.ads:7:21: medium: contract case might fail
show_failed_proof_attempt.ads:8:21: medium: contract case might fail
gnatprove: unproved check messages considered as errors

Runtime output

raised ADAASSERTIONSASSERTION_ERROR: failed contract case at show_failed_proof_attempt.ads:8

The error message shows that the first contract case is failing, which means that Incremented is True. However, if we print the value of Incremented before returning, we see that it’s False, as expected for the input we provided. The error here is that guards of contract cases are evaluated before the call, so our specification is wrong! To correct this, we should either write X < 1000 as the guard of the first case or use a standard postcondition with an if-expression.

58 Chapter 3. Proof of Program Integrity
3.4.2 Debugging Cases where more Information is Required

Even if both the code and the assertions are correct, GNATprove may still report that it can't prove a verification condition for a property. This can happen for two reasons:

- The property may be unprovable because the code is missing some assertion. One category of these cases is due to the modularity of the analysis which, as we discussed above, means that GNATprove only knows about the properties of your subprograms that you have explicitly written.
- There may be some information missing in the logical model of the program used by GNATprove.

Let's look at the case where the code and the specification are correct but there's some information missing. As an example, GNATprove finds the postcondition of `Increase` to be unprovable.

Listing 16: show_failed_proof_attempt.ads

```ada
package Show_Failed_Proof_Attempt is

   C : Natural := 100;

   procedure Increase (X : in out Natural) with
       Post => (if X'Old < C then X > X'Old else X = C);

end Show_Failed_Proof_Attempt;
```

Listing 17: show_failed_proof_attempt.adb

```ada
package body Show_Failed_Proof_Attempt is

   procedure Increase (X : in out Natural) is
       begin
           if X < 90 then
               X := X + 10;
           elsif X >= C then
               X := C;
           else
               X := X + 1;
           end if;
       end Increase;

end Show_Failed_Proof_Attempt;
```

This postcondition is a conditional. It says that if the parameter (X) is less than a certain value (C), its value will be increased by the procedure while if it's greater, its value will be set to C (saturated). When C has the value 100, the code of `Increases` adds 10 to the value of X if it was initially less than 90, increments X by 1 if it was between 90 and 99, and sets
X to 100 if it was greater or equal to 100. This behavior does satisfy the postcondition, so why is the postcondition not provable?

The values in the counterexample returned by GNATprove in its message gives us a clue: $C = 0$ and $X = 10$ and $X' \text{Old} = 0$. Indeed, if C is not equal to 100, our reasoning above is incorrect: the values of 0 for C and X on entry indeed result in X being 10 on exit, which violates the postcondition!

We probably didn't expect the value of C to change, or at least not to go below 90. But, in that case, we should have stated so by either declaring C to be constant or by adding a precondition to the Increase subprogram. If we do either of those, GNATprove is able to prove the postcondition.

### 3.4.3 Debugging Prover Limitations

Finally, there are cases where GNATprove provides a perfectly valid verification condition for a property, but it's nevertheless not proved by the automatic prover that runs in the later stages of the tool's execution. This is quite common. Indeed, GNATprove produces its verification conditions in first-order logic, which is not decidable, especially in combination with the rules of arithmetic. Sometimes, the automatic prover just needs more time. Other times, the prover will abandon the search almost immediately or loop forever without reaching a conclusive answer (either a proof or a counterexample).

For example, the postcondition of our GCD function below — which calculates the value of the GCD of two positive numbers using Euclidean's algorithm — can't be verified with GNATprove's default settings.

#### Listing 18: show_failed_proof_attempt.ads

```ada
package Show_Failed_Proof_Attempt is
  function GCD (A, B : Positive) return Positive with Post =>
    A mod GCD'Result = 0
    and B mod GCD'Result = 0;
end Show_Failed_Proof_Attempt;
```

#### Listing 19: show_failed_proof_attempt.adb

```ada
package body Show_Failed_Proof_Attempt is
  function GCD (A, B : Positive) return Positive is
    begin
      if A > B then
        return GCD (A - B, B);
      elsif B > A then
        return GCD (A, B - A);
      else
        return A;
      end if;
    end GCD;
end Show_Failed_Proof_Attempt;
```

---

**Code block metadata**

Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Failed_Proof_Attempt_4  
MD5: a6f1a39ceb0793df8a08691d59a5d9ce  

**Chapter 3. Proof of Program Integrity**
Introduction to SPARK

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_failed_proof_attempt.ads:5:08: medium: postcondition might fail, cannot prove
A mod GCD'Result = 0
 gnatprove: unproved check messages considered as errors

The first thing we try is increasing the amount of time the prover is allowed to spend on
each verification condition using the --timeout option of GNATprove (e.g., by using the
dialog box in GNAT Studio). In this example, increasing it to one minute, which is relatively
high, doesn't help. We can also specify an alternative automatic prover — if we have one
— using the option --prover of GNATprove (or the dialog box). For our postcondition, we
tried Alt-Ergo, cvc5, and Z3 without any luck.

Listing 20: show_failed_proof_attempt.ads

```adl
package Show_Failed_Proof_Attempt is
  function GCD (A, B : Positive) return Positive with
   Post =>
      A mod GCD'Result = 0
      and B mod GCD'Result = 0;
end Show_Failed_Proof_Attempt;
```

Listing 21: show_failed_proof_attempt.adb

```adl
package body Show_Failed_Proof_Attempt is
  function GCD (A, B : Positive) return Positive
  is
    Result : Positive;
  begin
    if A > B then
      Result := GCD (A - B, B);
      pragma Assert ((A - B) mod Result = 0);
      -- info: assertion proved
      pragma Assert (B mod Result = 0);
      -- info: assertion proved
      pragma Assert (A mod Result = 0);
      -- medium: assertion might fail
    elsif B > A then
      Result := GCD (A, B - A);
      pragma Assert ((B - A) mod Result = 0);
      -- info: assertion proved
    else
      Result := A;
    end if;
  return Result;
end GCD;
end Show_Failed_Proof_Attempt;
```

3.4. Debugging Failed Proof Attempts
To better understand the reason for the failure, we added intermediate assertions to simplify the proof and pin down the part that’s causing the problem. Adding such assertions is often a good idea when trying to understand why a property is not proved. Here, provers can’t verify that if both \( A - B \) and \( B \) can be divided by \( \text{Result} \), so can \( A \). This may seem surprising, but non-linear arithmetic, involving, for example, multiplication, modulo, or exponentiation, is a difficult topic for provers and is not handled very well in practice by any of the general-purpose ones like Alt-Ergo, cvc5, or Z3.

**Note:** For more details on how to investigate unproved checks, see the SPARK User’s Guide\(^{20}\).

### 3.5 Code Examples / Pitfalls

We end with some code examples and pitfalls.

#### 3.5.1 Example #1

The package Lists defines a linked-list data structure. We call `Link(I, J)` to make a link from index \( I \) to index \( J \) and call `Goes_To(I, J)` to determine if we’ve created a link from index \( I \) to index \( J \). The postcondition of `Link` uses `Goes_To` to state that there must be a link between its arguments once `Link` completes.

**Listing 22: lists.ads**

```ada
package Lists with SPARK_Mode is

  type Index is new Integer;

  function Goes_To (I, J : Index) return Boolean;

  procedure Link (I, J : Index) with Post => Goes_To (I, J);

private
```

---

type Cell (Is_Set : Boolean := True) is record
  case Is_Set is
  when True =>
    Next : Index;
  when False =>
    null;
  end case;
end record;
type Cell_Array is array (Index) of Cell;
Memory : Cell_Array;
end Lists;

package body Lists with SPARK_Mode is
  function Goes_To (I, J : Index) return Boolean is
    begin
      if Memory (I).Is_Set then
        Memory (I).Next = J;
      end if;
      return False;
    end Goes_To;
  procedure Link (I, J : Index) is
    begin
      Memory (I) := (Is_Set => True, Next => J);
    end Link;
end Lists;

This example is correct, but can't be verified by GNATprove. This is because Goes_To itself has no postcondition, so nothing is known about its result.
### 3.5.2 Example #2

We now redefine \texttt{Goes\_To} as an expression function.

#### Listing 24: lists.ads

```ada
package Lists with SPARK_Mode is

  type Index is new Integer;

  function Goes\_To (I, J : Index) return Boolean;

  procedure Link (I, J : Index) with Post => Goes\_To (I, J);

private

  type Cell (Is\_Set : Boolean := True) is record
    case Is\_Set is
      when True =>
        Next : Index;
      when False =>
        null;
    end case;
  end record;

  type Cell\_Array is array (Index) of Cell;

  Memory : Cell\_Array;

  function Goes\_To (I, J : Index) return Boolean is
    (Memory (I).Is\_Set and then Memory (I).Next = J);

end Lists;
```

#### Listing 25: lists.adb

```ada
package body Lists with SPARK_Mode is

  procedure Link (I, J : Index) is
  begin
    Memory (I) := (Is\_Set => True, Next => J);
  end Link;

end Lists;
```

#### Code block metadata

Project: Courses.Intro\_To\_Spark.Proof\_of\_Program\_Integrity.Example\_02
MD5: c65953bbe8a5f9fb77a4d94e2dd875f9

#### Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
lists.adb:5:18: info: discriminant check proved
lists.ads:7:47: info: postcondition proved
lists.ads:25:44: info: discriminant check proved

GNATprove can fully prove this version: \texttt{Goes\_To} is an expression function, so its body is available for proof (specifically, for creating the postcondition needed for the proof).
### 3.5.3 Example #3

The package Stacks defines an abstract stack type with a Push procedure that adds an element at the top of the stack and a function Peek that returns the content of the element at the top of the stack (without removing it).

**Listing 26: stacks.ads**

```ada
package Stacks with SPARK_Mode is

   type Stack is private;

   function Peek (S : Stack) return Natural;
   procedure Push (S : in out Stack; E : Natural) with
      Post => Peek (S) = E;

private

   Max : constant := 10;

   type Stack_Array is array (1 .. Max) of Natural;

   type Stack is record
      Top : Positive;
      Content : Stack_Array;
      end record;

   function Peek (S : Stack) return Natural is
      (if S.Top in S.Content'Range then S.Content (S.Top) else 0);
   end Stacks;
```

**Listing 27: stacks.adb**

```ada
package body Stacks with SPARK_Mode is

   procedure Push (S : in out Stack; E : Natural) is
      begin
         if S.Top >= Max then
            return;
         end if;
         S.Top := S.Top + 1;
         S.Content (S.Top) := E;
      end Push;
end Stacks;
```

**Code block metadata**

- **Project:** Courses.Intro_To_Spark.Proof_of_Program_Integrity.Example_03
- **MD5:** 917d624916c5ef14c4e454d6c56414fd

**Prover output**

- Phase 1 of 2: generation of Global contracts ...
- Phase 2 of 2: flow analysis and proof ...
- stacks.ads:7:14: medium: postcondition might fail
- gnatprove: unproved check messages considered as errors

This example isn't correct. The postcondition of Push is only satisfied if the stack isn't full when we call Push.
3.5.4 Example #4

We now change the behavior of Push so it raises an exception when the stack is full instead of returning.

Listing 28: stacks.ads

```ada
package Stacks with SPARK_Mode is

  type Stack is private;
  Is_Full_E : exception;

  function Peek (S : Stack) return Natural;
  procedure Push (S : in out Stack; E : Natural) with
  Post => Peek (S) = E;

private

  Max : constant := 10;

  type Stack_Array is array (1 .. Max) of Natural;

  type Stack is record
      Top : Positive;
      Content : Stack_Array;
  end record;

  function Peek (S : Stack) return Natural is
    (if S.Top in S.Content'Range then S.Content (S.Top) else 0);
end Stacks;
```

Listing 29: stacks.adb

```ada
package body Stacks with SPARK_Mode is

  procedure Push (S : in out Stack; E : Natural) is
  begin
    if S.Top >= Max then
      raise Is_Full_E;
    end if;
    S.Top := S.Top + 1;
    S.Content (S.Top) := E;
  end Push;
end Stacks;
```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Example_04
MD5: b573ebe93f85ea171166b6953cbb8956

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
stacks.adb:6:10: medium: exception might be raised
gnatprove: unproved check messages considered as errors

The postcondition of Push is now proved because GNATprove only considers execution paths
leading to normal termination. But it issues a message warning that exception Is_Full_E may be raised at runtime.

### 3.5.5 Example #5

Let’s add a precondition to Push stating that the stack shouldn’t be full.

```
Listing 30: stacks.ads
package Stacks with SPARK_Mode is
  type Stack is private;
    Is_Full_E : exception;
  function Peek (S : Stack) return Natural;
  function Is_Full (S : Stack) return Boolean;
  procedure Push (S : in out Stack; E : Natural) with
    Pre => not Is_Full (S),
    Post => Peek (S) = E;
private
  Max : constant := 10;
  type Stack_Array is array (1 .. Max) of Natural;
  type Stack is record
    Top : Positive;
    Content : Stack_Array;
  end record;
  function Peek (S : Stack) return Natural is
    (if S.Top in S.Content’Range then S.Content (S.Top) else 0);
  function Is_Full (S : Stack) return Boolean is (S.Top >= Max);
end Stacks;
```

```
Listing 31: stacks.adb
package body Stacks with SPARK_Mode is
  procedure Push (S : in out Stack; E : Natural) is
    begin
      if S.Top >= Max then
        raise Is_Full_E;
      end if;
      S.Top := S.Top + 1;
      S.Content (S.Top) := E;
    end Push;
end Stacks;
```

**Code block metadata**

*Project: Courses.Intro.To_Spark.Proof_of_Program_Integrity.Example_05*
*MD5: 63c2df0d680d5ac6c91d8d49726e7423e*
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
stacks.adb:6:10: info: raise statement or expression proved unreachable
stacks.adb:8:22: info: overflow check proved
stacks.adb:9:19: info: index check proved
stacks.ads:11:14: info: postcondition proved
stacks.ads:25:52: info: index check proved

This example is correct. With the addition of the precondition, GNATprove can now verify that \texttt{Is\_Full\_E} can never be raised at runtime.

### 3.5.6 Example #6

The package \texttt{Memories} defines a type \texttt{Chunk} that models chunks of memory. Each element of the array, represented by its index, corresponds to one data element. The procedure \texttt{Read\_Record} reads two pieces of data starting at index \texttt{From} out of the chunk represented by the value of \texttt{Memory}.

#### Listing 32: memories.ads

```ada
package Memories is
  type Chunk is array (Integer range <>) of Integer
    with Predicate => Chunk'Length >= 10;
  function Is_Too_Coarse (V : Integer) return Boolean;
  procedure Treat_Value (V : out Integer);
end Memories;
```

#### Listing 33: read_record.adb

```ada
with Memories; use Memories;

procedure Read_Record (Memory : Chunk; From : Integer)
  with SPARK Mode => On,
    Pre => From in Memory'First .. Memory'Last - 2
  is
    function Read_One (First : Integer; Offset : Integer) return Integer
      with Pre => First + Offset in Memory'Range
    is
      Value : Integer := Memory (First + Offset);
    begin
      if Is_Too_Coarse (Value) then
        Treat_Value (Value);
      end if;
      return Value;
    end Read_One;

  Datal, Data2 : Integer;

  begin
    Datal := Read_One (From, 1);
    Data2 := Read_One (From, 2);
  end Read_Record;
```

**Code block metadata**
Project: Courses.Intro_To_Spark.Proof_of_Program_Integrity.Example_06
MD5: aec8014dc291708999092fa123ee7416

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
read_record.adb:8:24: medium: overflow check might fail, cannot prove lower bound
for First + Offset [reason for check: result of addition must fit in a 32-bits machine integer] [possible fix: use pragma Overflow_Mode or switch -gnato13 or unit Ada.Numerics.Big_Numerics.Big_Integers]
gnatprove: unproved check messages considered as errors

This example is correct, but it can't be verified by GNATprove, which analyses Read_One on its own and notices that an overflow may occur in its precondition in certain contexts.

3.5.7 Example #7

Let's rewrite the precondition of Read_One to avoid any possible overflow.

Listing 34: memories.ads

```ada
package Memories is
  type Chunk is array (Integer range <>) of Integer
  with Predicate => Chunk'Length >= 10;
  function Is_Too_Coarse (V : Integer) return Boolean;
  procedure Treat_Value (V : out Integer);
end Memories;
```

Listing 35: read_record.adb

```ada
with Memories; use Memories;

procedure Read_Record (Memory : Chunk; From : Integer)
  with SPARK_Mode => On,
  Pre => From in Memory'First .. Memory'Last - 2
is
  function Read_One (First : Integer; Offset : Integer) return Integer
  with Pre => First >= Memory'First
  and then Offset in 0 .. Memory'Last - First
  is
    Value : Integer := Memory (First + Offset);
    begin
      if Is_Too_Coarse (Value) then
        Treat_Value (Value);
      end if;
      return Value;
    end Read_One;

  Datal, Data2 : Integer;

begin
  Datal := Read_One (From, 1);
  Data2 := Read_One (From, 2);
end Read_Record;
```
Introduction to SPARK

3.5.8 Example #8

Let’s completely remove the precondition of Read_One.

Listing 36: memories.ads

```ada
package Memories is

  type Chunk is array (Integer range <>) of Integer
    with Predicate => Chunk'Length >= 10;

  function Is_Too_Coarse (V : Integer) return Boolean;

  procedure Treat_Value (V : out Integer);

end Memories;
```

Listing 37: read_record.adb

```ada
with Memories; use Memories;

procedure Read_Record (Memory : Chunk; From : Integer)
  with SPARK_Mode => On,
    Pre => From in Memory'First .. Memory'Last - 2
is
  function Read_One (First : Integer; Offset : Integer) return Integer is
    Value : Integer := Memory (First + Offset);
  begin
    if Is_Too_Coarse (Value) then
      Treat_Value (Value);
    end if;
    return Value;
  end Read_One;

  Data1, Data2 : Integer;

begin
  Data1 := Read_One (From, 1);
  Data2 := Read_One (From, 2);
end Read_Record;
```
3.5.9 Example #9

The procedure Compute performs various computations on its argument. The computation performed depends on its input range and is reflected in its contract, which we express using a Contract_Cases aspect.

Listing 38: compute.adb

```
procedure Compute (X : in out Integer) with
  Contract_Cases => ((X in -100 .. 100) => X = X'Old * 2,
   (X in 0 .. 199) => X = X'Old + 1,
   (X in -199 .. 0) => X = X'Old - 1,
   X >= 200 => X = 200,
   others => X = -200)
is
begin
  if X in -100 .. 100 then
    X := X * 2;
  elsif X in 0 .. 199 then
    X := X + 1;
  elsif X in -199 .. 0 then
    X := X - 1;
  elsif X >= 200 then
    X := 200;
  else
    X := -200;
  end if;
end Compute;
```

This example is correct and fully proved. We could have fixed the contract of Read_One to correctly handle both positive and negative values of Memory'Last, but we found it simpler to let the function be inlined for proof by removing its precondition.
Introduction to SPARK

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
compute.adb:2:03: medium: contract cases might not be disjoint
compute.adb:3:41: medium: contract case might fail
compute.adb:4:41: medium: contract case might fail
gnatprove: unproved check messages considered as errors

This example isn't correct. We duplicated the content of Compute's body in its contract. This is incorrect because the semantics of Contract_Cases require disjoint cases, just like a case statement. The counterexample returned by GNATprove shows that \( X = 0 \) is covered by two different case-guards (the first and the second).

### 3.5.10 Example #10

Let's rewrite the contract of Compute to avoid overlapping cases.

```ada
procedure Compute (X : in out Integer) with
  Contract_Cases => ((X in 0 .. 199) => X >= X'Old,
                     (X in -199 .. -1) => X <= X'Old,
                     X >= 200 => X = 200,
                     X < -200 => X = -200)
  is
begin
  if X in -100 .. 100 then
    X := X * 2;
  elsif X in 0 .. 199 then
    X := X + 1;
  elsif X in -199 .. 0 then
    X := X - 1;
  elsif X >= 200 then
    X := 200;
  else
    X := -200;
  end if;
end Compute;
```

This example is still not correct. GNATprove can successfully prove the different cases are disjoint and also successfully verify each case individually. This isn't enough, though: a Contract_Cases must cover all cases. Here, we forgot the value -200, which is what GNATprove reports in its counterexample.
Abstraction is a key concept in programming that can drastically simplify both the implementation and maintenance of code. It's particularly well suited to SPARK and its modular analysis. This section explains what state abstraction is and how you use it in SPARK. We explain how it impacts GNATprove's analysis both in terms of information flow and proof of program properties.

State abstraction allows us to:

- express dependencies that wouldn't otherwise be expressible because some data that's read or written isn't visible at the point where a subprogram is declared — examples are dependencies on data, for which we use the Global contract, and on flow, for which we use the Depends contract.
- reduce the number of variables that need to be considered in flow analysis and proof, a reduction which may be critical in order to scale the analysis to programs with thousands of global variables.

4.1 What's an Abstraction?

Abstraction is an important part of programming language design. It provides two views of the same object: an abstract one and a refined one. The abstract one — usually called specification — describes what the object does in a coarse way. A subprogram's specification usually describes how it should be called (e.g., parameter information such as how many and of what types) as well as what it does (e.g., returns a result or modifies one or more of its parameters).

Contract-based programming, as supported in Ada, allows contracts to be added to a subprogram's specification. You use contracts to describe the subprogram's behavior in a more fine-grained manner, but all the details of how the subprogram actually works are left to its refined view, its implementation.

Take a look at the example code shown below.

Listing 1: increase.ads

```ada
procedure Increase (X : in out Integer) with
  Global => null,
  Pre    => X <= 100,
  Post   => X'Old < X;
```

Listing 2: increase.adb

```ada
procedure Increase (X : in out Integer) is
begin
  X := X + 1;
end Increase;
```
We’ve written a specification of the subprogram Increase to say that it’s called with a single argument, a variable of type \texttt{Integer} whose initial value is less than 100. Our contract says that the only effect of the subprogram is to increase the value of its argument.

### 4.2 Why is Abstraction Useful?

A good abstraction of a subprogram’s implementation is one whose specification precisely and completely summarizes what its callers can rely on. In other words, a caller of that subprogram shouldn't rely on any behavior of its implementation if that behavior isn't documented in its specification.

For example, callers of the subprogram Increase can assume that it always strictly increases the value of its argument. In the code snippet shown below, this means the loop must terminate.

```plaintext
Listing 3: increase.ads

1 procedure Increase (X : in out Integer) with
2   Global => null,
3   Pre   => X <= 100,
4   Post  => X'old < X;
```

```plaintext
Listing 4: client.adb

1 with Increase;
2 procedure Client is
3   X : Integer := 0;
4 begin
5   while X <= 100 loop -- The loop will terminate
6     Increase (X); -- Increase can be called safely
7   end loop;
8   pragma Assert (X = 101); -- Will this hold?
9 end Client;
```

---

The Prover output indicates that the assertion in the client.adb file might fail, with gnatprove considering unproved check messages as errors.
Callers can also assume that the implementation of Increase won't cause any runtime errors when called in the loop. On the other hand, nothing in the specification guarantees that the assertion shown above is correct: it may fail if Increase's implementation is changed.

If you follow this basic principle, abstraction can bring you significant benefits. It simplifies both your program's implementation and verification. It also makes maintenance and code reuse much easier since changes to the implementation of an object shouldn't affect the code using this object. Your goal in using it is that it should be enough to understand the specification of an object in order to use that object, since understanding the specification is usually much simpler than understanding the implementation.

GNATprove relies on the abstraction defined by subprogram contracts and therefore doesn't prove the assertion after the loop in Client above.

### 4.3 Abstraction of a Package's State

Subprograms aren't the only objects that benefit from abstraction. The state of a package — the set of persistent variables defined in it — can also be hidden from external users. You achieve this form of abstraction — called state abstraction — by defining variables in the body or private part of a package so they can only be accessed through subprogram calls. For example, our Stack package shown below provides an abstraction for a Stack object which can only be modified using the Pop and Push procedures.

```gherkin
package Stack is
  procedure Pop (E : out Element);
  procedure Push (E : in Element);
end Stack;

package body Stack is
  Content : Element_Array (1 .. Max);
  Top : Natural;
end Stack;
```

The fact that we implemented it using an array is irrelevant to the caller. We could change that without impacting our callers' code.

### 4.4 Declaring a State Abstraction

Hidden state influences a program's behavior, so SPARK allows that state to be declared. You can use the Abstract_State aspect, an abstraction that names a state, to do this, but you aren't required to use it even for a package with hidden state. You can use several state abstractions to declare the hidden state of a single package or you can use it for a package with no hidden state at all. However, since SPARK doesn't allow aliasing, different state abstractions must always refer to disjoint sets of variables. A state abstraction isn't a variable: it doesn't have a type and can't be used inside expressions, either those in bodies or contracts.

As an example of the use of this aspect, we can optionally define a state abstraction for the entire hidden state of the Stack package like this:

```gherkin
package Stack with
  Abstract_State => The_Stack
is
...
```

Alternatively, we can define a state abstraction for each hidden variable.
Introduction to SPARK

**package Stack with**

  **Abstract_State** => (Top_State, Content_State)

**is**

...

Remember: a state abstraction isn't a variable (it has no type) and can't be used inside expressions. For example:

```plaintext```
pragma Assert (Stack.Top_State = ...);
-- compilation error: Top_State is not a variable
```

### 4.5 Refining an Abstract State

Once you've declared an abstract state in a package, you must refine it into its constituents using a **Refined_State** aspect. You must place the **Refined_State** aspect on the package body even if the package wouldn't otherwise have required a body. For each state abstraction you've declared for the package, you list the set of variables represented by that state abstraction in its refined state.

If you specify an abstract state for a package, it must be complete, meaning you must have listed every hidden variable as part of some state abstraction. For example, we must add a **Refined_State** aspect on our **Stack** package's body linking the state abstraction (**The_Stack**) to the entire hidden state of the package, which consists of both **Content** and **Top**.

Listing 5: stack.ads

```plaintext```
```ada```
package Stack with
  Abstract_State => The_Stack
is
  type Element is new Integer;
  procedure Pop (E : out Element);
  procedure Push (E : Element);
end Stack;
```ada```
```plaintext```
Listing 6: stack.adb

```plaintext```
```ada```
package body Stack with
  Refined_State => (The_Stack => (Content, Top))
is
  Max : constant := 100;
  type Element_Array is array (1 .. Max) of Element;
  Content : Element_Array := (others => 0);
  Top : Natural range 0 .. Max := 0;
  -- Both Content and Top must be listed in the list of constituents of The_Stack

  procedure Pop (E : out Element) is
  begin
    E := Content (Top);
    Top := Top - 1;
  end Pop;

  procedure Push (E : Element) is
```

(continues on next page)
begin
  Top := Top + 1;
  Content (Top) := E;
end Push;

end Stack;

4.6 Representing Private Variables

You can refine state abstractions in the package body, where all the variables are visible. When only the package's specification is available, you need a way to specify which state abstraction each private variable belongs to. You do this by adding the Part_Of aspect to the variable's declaration.

Part_Of annotations are mandatory: if you gave a package an abstract state annotation, you must link all the hidden variables defined in its private part to a state abstraction. For example:

Listing 7: stack.ads

```ada
package Stack with
  Abstract_State => The_Stack
is
  type Element is new Integer;

  procedure Pop (E : out Element);
  procedure Push (E : Element);

private
  Max : constant := 100;

  type Element_Array is array (1 .. Max) of Element;

  Content : Element_Array with Part_Of => The_Stack;
  Top : Natural range 0 .. Max with Part_Of => The_Stack;
end Stack;
```

4.6. Representing Private Variables
Since we chose to define Content and Top in Stack's private part instead of its body, we had to add a Part_Of aspect to both of their declarations, associating them with the state abstraction The_Stack, even though it's the only state abstraction. However, we still need to list them in the Refined_State aspect in Stack's body.

```verbatim
package body Stack with
   Refined_State => (The_Stack => (Content, Top))
```

## 4.7 Additional State

### 4.7.1 Nested Packages

So far, we've only discussed hidden variables. But variables aren't the only component of a package's state. If a package \( P \) contains a nested package, the nested package's state is also part of \( P \)'s state. If the nested package is hidden, its state is part of \( P \)'s hidden state and must be listed in \( P \)'s state refinement.

We see this in the example below, where the package Hidden_Nested's hidden state is part of \( P \)'s hidden state.

### Listing 8: p.ads

```verbatim
package P with
   Abstract_State => State
is
   package Visible_Nested with
      Abstract_State => Visible_State
   is
      procedure Get (E : out Integer);
   end Visible_Nested;
end P;
```

### Listing 9: p.adb

```verbatim
package body P with
   Refined_State => (State => Hidden_Nested.Hidden_State)
is
   package Hidden_Nested with
      Abstract_State => Hidden_State,
      Initializes => Hidden_State
   is
      function Get return Integer;
   end Hidden_Nested;
end P;

decomp
```

```verbatim
package body Hidden_Nested with
   Refined_State => (Hidden_State => Cnt)
is
   Cnt : Integer := 0;
   function Get return Integer is (Cnt);
end Hidden_Nested;

decomp
```

```verbatim
package body Visible_Nested with
   Refined_State => (Visible_State => Checked)
```

(continues on next page)
is

    Checked : Boolean := False;

procedure Get (E : out Integer) is
begin
    Checked := True;
    E := Hidden_Nested.Get;
end Get;
end Visible_Nested;
end P;

4.7.2 Constants that Depend on Variables

Some constants are also possible components of a state abstraction. These are constants
whose value depends either on a variable or a subprogram parameter. They're handled as
variables during flow analysis because they participate in the flow of information between
variables throughout the program. Therefore, GNATprove considers these constants to be
part of a package's state just like it does for variables.

If you've specified a state abstraction for a package, you must list such hidden constants
declared in that package in the state abstraction refinement. However, constants that don't
depend on variables don't participate in the flow of information and must not appear in a
state refinement.

Let's look at this example.

Listing 10: stack.ads

package Stack with
  Abstract_State => The_Stack
is
  type Element is new Integer;

procedure Pop (E : out Element);
procedure Push (E : Element);
end Stack;

Listing 11: configuration.ads

package Configuration with
  Initializes => External_Variable
is
  (continues on next page)
External_Variable : Positive with Volatile;
end Configuration;

Listing 12: stack.adb

with Configuration;
pragma Elaborate (Configuration);

package body Stack with
  Refined_State => (The_Stack => (Content, Top, Max))
-- Max has variable inputs. It must appear as a
-- constituent of The_Stack
is
  Max : constant Positive := Configuration.External_Variable;

  type Element_Array is array (1 .. Max) of Element;
  Content : Element_Array := (others => 0);
  Top : Natural range 0 .. Max := 0;

  procedure Pop (E : out Element) is
  begin
    E := Content (Top);
    Top := Top - 1;
  end Pop;

  procedure Push (E : Element) is
  begin
    Top := Top + 1;
    Content (Top) := E;
  end Push;

end Stack;

Code block metadata

Project: Courses.Intro_To_Spark.State_Abstraction.Const\ntants_And_Variables
MD5: 109a6340ef0f3b0dc88e0fe5888b9a53

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
stack.ads:6:20: info: initialization of "E" proved
configuration.ads:2:03: info: flow dependencies proved

Here, Max — the maximum number of elements that can be stored in the stack — is initialized from a variable in an external package. Because of this, we must include Max as part of the state abstraction The_Stack.

Note: For more details on state abstractions, see the SPARK User's Guide\n[21].

### 4.8 Subprogram Contracts

#### 4.8.1 Global and Depends

Hidden variables can only be accessed through subprogram calls, so you document how state abstractions are modified during the program’s execution via the contracts of those subprograms. You use Global and Depends contracts to specify which of the state abstractions are used by a subprogram and how values flow through the different variables. The Global and Depends contracts that you write when referring to state abstractions are often less precise than contracts referring to visible variables since the possibly different dependencies of the hidden variables contained within a state abstraction are collapsed into a single dependency.

Let's add Global and Depends contracts to the Pop procedure in our stack.

```ada
package Stack with
  Abstract_State => (Top_State, Content_State)
is
  type Element is new Integer;

procedure Pop (E : out Element) with
  Global => (Input => Content_State,
             In_Out => Top_State),
  Depends => (Top_State => Top_State,
              E => (Content_State, Top_State));
end Stack;
```

**Code block metadata**

**Project**: Courses.Intro_To_Spark.State_Abstraction.Global_Depends  
**MD5**: a7b383c35508d6a8294bf7cf0fe332ac

**Prover output**

Phase 1 of 2: generation of Global contracts ...  
Phase 2 of 2: analysis of data and information flow ...

In this example, the Pop procedure only modifies the value of the hidden variable Top, while Content is unchanged. By using distinct state abstractions for the two variables, we’re able to preserve this semantic in the contract.

Let’s contrast this example with a different representation of Global and Depends contracts, this time using a single abstract state.

```ada
package Stack with
  Abstract_State => The_Stack
is
  type Element is new Integer;

procedure Pop (E : out Element) with
  Global => (In_Out => The_Stack),
  Depends => ((The_Stack, E) => The_Stack);
end Stack;
```

**Code block metadata**
Here, Top_State and Content_State are merged into a single state abstraction, The_Stack. By doing so, we've hidden the fact that Content isn't modified (though we're still showing that Top may be modified). This loss in precision is reasonable here, since it's the whole point of the abstraction. However, you must be careful not to aggregate unrelated hidden state because this risks their annotations becoming meaningless.

Even though imprecise contracts that consider state abstractions as a whole are perfectly reasonable for users of a package, you should write Global and Depends contracts that are as precise as possible within the package body. To allow this, SPARK introduces the notion of **refined contracts**, which are precise contracts specified on the bodies of subprograms where state refinements are visible. These contracts are the same as normal Global and Depends contracts except they refer directly to the hidden state of the package.

When a subprogram is called inside the package body, you should write refined contracts instead of the general ones so that the verification can be as precise as possible. However, refined Global and Depends are optional: if you don't specify them, GNATprove will compute them to check the package's implementation.

For our Stack example, we could add refined contracts as shown below.

```ada
package Stack with
    Abstract_State => The_Stack
is
    type Element is new Integer;

    procedure Pop (E : out Element) with
        Global => (In_Out => The_Stack),
        Depends => ((The_Stack, E) => The_Stack);

    procedure Push (E : Element) with
        Global => (In_Out => The_Stack),
        Depends => (The_Stack => (The_Stack, E));
end Stack;
```

```ada
package body Stack with
    Refined_State => (The_Stack => (Content, Top))
is
    Max : constant := 100;

    type Element_Array is array (1 .. Max) of Element;

    Content : Element_Array := (others => 0);
    Top : Natural range 0 .. Max := 0;

    procedure Pop (E : out Element) with
        Refined_Global => (Input => Content,
                           In_Out => Top),
        Refined_Depends => (Top => Top,
```


```ada
E => (Content, Top))

begin
  E := Content (Top);
  Top := Top - 1;
end Pop;

procedure Push (E : Element) with
  Pre => not Is_Empty,
  Post => not Is_Full;
begin
  Top := Top + 1;
  Content (Top) := E;
end Push;

end Stack;
```


### Code block metadata

Project: Courses.Intro_To_Spark.State_abstraction.global_refined
MD5: b7e700645885155ea7faf2f4170f0462

#### Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...

### 4.8.2 Preconditions and Postconditions

We mostly express functional properties of subprograms using preconditions and postconditions. These are standard Boolean expressions, so they can't directly refer to state abstractions. To work around this restriction, we can define functions to query the value of hidden variables. We then use these functions in place of the state abstraction in the contract of other subprograms.

For example, we can query the state of the stack with functions `Is_Empty` and `Is_Full` and call these in the contracts of procedures `Pop` and `Push`:

```ada
package Stack is
  type Element is new Integer;
  function Is_Empty return Boolean;
  function Is_Full return Boolean;
  procedure Pop (E : out Element) with
    Pre => not Is_Empty,
    Post => not Is_Full;

  procedure Push (E : Element) with
    Pre => not Is_Full,
    Post => not Is_Empty;

end Stack;
```

### 4.8. Subprogram Contracts
Introduction to SPARK

Listing 18: stack.adb

```ada
package body Stack is

  Max : constant := 100;

  type Element_Array is array (1 .. Max) of Element;

  Content : Element_Array := (others => 0);
  Top : Natural range 0 .. Max := 0;

  function Is_Empty return Boolean is (Top = 0);
  function Is_Full return Boolean is (Top = Max);

  procedure Pop (E : out Element) is
  begin
    E := Content (Top);
    Top := Top - 1;
  end Pop;

  procedure Push (E : Element) is
  begin
    Top := Top + 1;
    Content (Top) := E;
  end Push;

end Stack;
```

Code block metadata

Project: Courses.Intro_To_Spark.State_Abstraction.Pre_Postconditions_1
MD5: fe9d4b65balbeeabc7cf0fed29b8b3c

Prover output

```
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
stack.adb:15:23: info: index check proved
stack.adb:16:18: info: range check proved
stack.adb:21:28: info: range check proved
stack.adb:22:16: info: index check proved
stack.ads:7:19: info: initialization of "E" proved
stack.ads:9:14: info: postcondition proved
stack.ads:13:14: info: postcondition proved
```

Just like we saw for Global and Depends contracts, you may often find it useful to have a more precise view of functional contracts in the context where the hidden variables are visible. You do this using expression functions in the same way we did for the functions Is_Empty and Is_Full above. As expression function, bodies act as contracts for GNATprove, so they automatically give a more precise version of the contracts when their implementation is visible.

You may often need a more constraining contract to verify the package's implementation but want to be less strict outside the abstraction. You do this using the Refined_Post aspect. This aspect, when placed on a subprogram's body, provides stronger guarantees to internal callers of a subprogram. If you provide one, the refined postcondition must imply the subprogram's postcondition. This is checked by GNATprove, which reports a failing postcondition if the refined postcondition is too weak, even if it's actually implied by the subprogram's body. SPARK doesn't perform a similar verification for normal preconditions.

For example, we can refine the postconditions in the bodies of Pop and Push to be more detailed than what we wrote for them in their specification.
Listing 19: stack.ads

```ada
package Stack is
  type Element is new Integer;
  function Is_Empty return Boolean;
  function Is_Full return Boolean;
  procedure Pop (E : out Element) with
    Pre => not Is_Empty,
    Post => not Is_Full;
  procedure Push (E : Element) with
    Pre => not Is_Full,
    Post => not Is_Empty;
end Stack;
```

Listing 20: stack.adb

```ada
package body Stack is
  Max : constant := 100;
  type Element_Array is array (1 .. Max) of Element;
  Content : Element_Array := (others => 0);
  Top : Natural range 0 .. Max := 0;
  function Is_Empty return Boolean is (Top = 0);
  function Is_Full return Boolean is (Top = Max);
  procedure Pop (E : out Element) with
    Refined_Post => not Is_Full and E = Content (Top)'Old
  is
    begin
      E := Content (Top);
      Top := Top - 1;
    end Pop;
  procedure Push (E : Element) with
    Refined_Post => not Is_Empty and E = Content (Top)
  is
    begin
      Top := Top + 1;
      Content (Top) := E;
    end Push;
end Stack;
```

Code block metadata

- Project: Courses.Intro_To_Spark.State_Abstraction.Pre_Postconditions_2
- MD5: 4691565d58ba039b3cbd06e65cecfa88

Prover output

- Phase 1 of 2: generation of Global contracts ...
- Phase 2 of 2: flow analysis and proof ...
- stack.adb:14:51: info: index check proved

(continues on next page)
4.9 Initialization of Local Variables

As part of flow analysis, GNATprove checks for the proper initialization of variables. Therefore, flow analysis needs to know which variables are initialized during the package's elaboration.

You can use the Initializes aspect to specify the set of visible variables and state abstractions that are initialized during the elaboration of a package. An Initializes aspect can't refer to a variable that isn't defined in the unit since, in SPARK, a package can only initialize variables declared immediately within the package.

Initializes aspects are optional. If you don't supply any, they'll be derived by GNATprove. For our Stack example, we could add an Initializes aspect.

```ada
package Stack is
    Abstract_State => The_Stack,
    Initializes   => The_Stack
is
    type Element is new Integer;
    procedure Pop (E : out Element);
end Stack;
```

Listing 22: stack.adb

```ada
package body Stack is
    Refined_State => (The_Stack => (Content, Top))
is
    Max         : constant := 100;
    type Element_Array is array (1 .. Max) of Element;
    Content     : Element_Array := (others => 0);
    Top         : Natural range 0 .. Max := 0;
    procedure Pop (E : out Element) is
      begin
        E := Content (Top);
    end Pop;
end Stack;
```

Note: For more details on refinement in contracts, see the SPARK User's Guide. 

Top := Top - 1;
end Pop;

end Stack;

4.9. Initialization of Local Variables 87
Introduction to SPARK

plete: you must list every initialized state of the package, along with all its external dependencies.

Note: For more details on Initializes, see the SPARK User’s Guide\(^\text{23}\).

4.10 Code Examples / Pitfalls

This section contains some code examples to illustrate potential pitfalls.

4.10.1 Example #1

Package Communication defines a hidden local package, Ring_Buffer, whose capacity is initialized from an external configuration during elaboration.

Listing 25: configuration.ads

```ada
package Configuration is
  External_Variable : Natural := 1;
end Configuration;
```

Listing 26: communication.ads

```ada
with Configuration;

package Communication with
  Abstract_State => State,
  Initializes => (State => Configuration.External_Variable)
is
  function Get_Capacity return Natural;

private

package Ring_Buffer with
  Initializes => (Capacity => Configuration.External_Variable)
is
  Capacity : constant Natural := Configuration.External_Variable;
end Ring_Buffer;
end Communication;
```

Listing 27: communication.adb

```ada
package body Communication with
  Refined_State => (State => Ring_Buffer.Capacity)
is
  function Get_Capacity return Natural is
  begin
    return Ring_Buffer.Capacity;
  end Get_Capacity;
```

9 end Communication;

**Code block metadata**

Project: Courses.Intro_To_Spark.State_Abstraction.Example_01
MD5: 207e999f05a5b39fa2b9aebbc836b479

**Prover output**

Phase 1 of 2: generation of Global contracts ...
communication.adb:2:41: error: "Capacity" cannot act as constituent of state "State"
communication.adb:2:41: error: missing Part_Of indicator at communication.ads:14,
should specify encapsulator "State"
gnatprove: error during generation of Global contracts

This example isn't correct. Capacity is declared in the private part of Communication. Therefore, we should have linked it to State by using the Part_Of aspect in its declaration.

### 4.10.2 Example #2

Let's add Part_Of to the state of hidden local package Ring_Buffer, but this time we hide variable Capacity inside the private part of Ring_Buffer.

Listing 28: configuration.ads

```ada
package Configuration is
  External_Variable : Natural := 1;
end Configuration;
```

Listing 29: communication.ads

```ada
with Configuration;

package Communication with
  Abstract_State => State
is
  private

    package Ring_Buffer with
      Abstract_State => (B_State with Part_Of => State),
      Initializes => (B_State => Configuration.External_Variable)
    is
      function Get_Capacity return Natural;
    private
      Capacity : constant Natural := Configuration.External_Variable
      with Part_Of => B_State;
    end Ring_Buffer;

end Communication;
```

Listing 30: communication.adb

```ada
package body Communication with
  Refined_State => (State => Ring_Buffer.B_State)
```

(continues on next page)
is

package body Ring_Buffer with
  Refined_State => (B_State => Capacity)
  is
    function Get_Capacity return Natural is (Capacity);
end Ring_Buffer;
end Communication;

Code block metadata

Project: Courses.Intro_To_Spark.State_Abstraction.Example_02
MD5: b8d31fcfbd11bf305646efe07baeb91b

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
communication.ads:10:06: info: flow dependencies proved

This program is correct and GNATprove is able to verify it.

4.10.3 Example #3

Package Counting defines two counters: Black.Counter and Red.Counter. It provides separate initialization procedures for each, both called from the main procedure.

Listing 31: counting.ads

package Counting with
  Abstract_State => State
  is
    procedure Reset_Black_Count;
    procedure Reset_Red_Count;
end Counting;

Listing 32: counting.adb

package body Counting with
  Refined_State => (State => (Black.Counter, Red.Counter))
  is
    Black.Counter, Red.Counter : Natural;
    procedure Reset_Black_Count is
      begin
        Black.Counter := 0;
        end Reset_Black_Count;
    procedure Reset_Red_Count is
      begin
        Red.Counter := 0;
        end Reset_Red_Count;
end Counting;
Listing 33: main.adb

```ada
with Counting; use Counting;

procedure Main is
begin
  Reset_Black_Count;
  Reset_Red_Count;
end Main;
```

**Code block metadata**

Project: Courses.Intro_To_Spark.State_Abstraction.Example_03
MD5: bc2d7ccd7419d34f7156a16dfc484229

**Prover output**

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
...of main program "Main"
counting.ads:2:21: warning: no procedure exists that can initialize abstract state "Counting.State"
gnatprove: unproved check messages considered as errors

This program doesn't read any uninitialized data, but GNATprove fails to verify that. This is because we provided a state abstraction for package Counting, so flow analysis computes the effects of subprograms in terms of this state abstraction and thus considers State to be an in-out global consisting of both Black_Counter and Red_Counter. So it issues the message requiring that State be initialized after elaboration as well as the warning that no procedure in package Counting can initialize its state.

### 4.10.4 Example #4

Let's remove the abstract state on package Counting.

Listing 34: counting.ads

```ada
package Counting is
  procedure Reset_Black_Count;
  procedure Reset_Red_Count;
end Counting;
```

Listing 35: counting.adb

```ada
package body Counting is
  Black_Counter, Red_Counter : Natural;

  procedure Reset_Black_Count is
  begin
    Black_Counter := 0;
  end Reset_Black_Count;

  procedure Reset_Red_Count is
  begin
    Red_Counter := 0;
  end Reset_Red_Count;
end Counting;
```
Introduction to SPARK

Listing 36: main.adb

with Counting; use Counting;

procedure Main is
begin
  Reset_Black_Count;
  Reset_Red_Count;
end Main;

Code block metadata

Project: Courses.Intro_To_Spark.State_Abstraction.Example_04
MD5: 3ddd934b6ede6df7b823e46828694d12

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...

This example is correct. Because we didn't provide a state abstraction, GNATprove reasons in terms of variables, instead of states, and proves data initialization without any problem.

4.10.5 Example #5

Let's restore the abstract state to package Counting, but this time provide a procedure Reset_All that calls the initialization procedures Reset_Black_Counter and Reset_Red_Counter.

Listing 37: counting.ads

package Counting with
  Abstract_State => State
is
  procedure Reset_Black_Count with Global => (In_Out => State);
  procedure Reset_Red_Count with Global => (In_Out => State);
  procedure Reset_All with Global => (Output => State);
end Counting;

Listing 38: counting.adb

package body Counting with
  Refined_State => (State => (Black_Counter, Red_Counter))
is
  Black_Counter, Red_Counter : Natural;

  procedure Reset_Black_Count is
  begin
    Black_Counter := 0;
  end Reset_Black_Count;

  procedure Reset_Red_Count is
  begin
    Red_Counter := 0;
  end Reset_Red_Count;

  procedure Reset_All is
  begin
    Reset_Black_Count;
    Reset_Red_Count;
  end Reset_All;

  (continues on next page)
Reset_Red_Count;
end Reset_All;
end Counting;

**Code block metadata**

Project: Courses.Intro_To_Spark.State_Abstraction.Example_05
MD5: d123ccc644fe6999699388708f2ecf89

**Prover output**

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
counting.ads:4:37: info: data dependencies proved
counting.ads:5:37: info: data dependencies proved
counting.ads:6:14: info: initialization of "Black_Counter" constituent of "State" proved
counting.ads:6:14: info: initialization of "Red_Counter" constituent of "State" proved
counting.ads:6:37: info: data dependencies proved

This example is correct. Flow analysis computes refined versions of Global contracts for internal calls and uses these to verify that Reset_All indeed properly initializes State. The Refined_Global and Global annotations are not mandatory and can be computed by GNATprove.

**4.10.6 Example #6**

Let's consider yet another version of our abstract stack unit.

```ada
package Stack with
Abstract_State => The_Stack
is
pragma Unevaluated_Use_Of_Old (Allow);

type Element is new Integer;

type Element_Array is array (Positive range <>) of Element;

Max : constant Natural := 100;

subtype Length_Type is Natural range 0 .. Max;

procedure Push (E : Element) with
Post =>
not Is_Empty and
(if Is_Full'Old then The_Stack = The_Stack'Old else Peek = E);

function Peek return Element with Pre => not Is_Empty;
function Is_Full return Boolean;
function Is_Empty return Boolean;
end Stack;
```

```ada
package body Stack with
Refined_State => (The_Stack => (Top, Content))
is
Top : Length_Type := 0;
```

(continues on next page)
Content : Element_Array (1 .. Max) := (others => 0);

procedure Push (E : Element) is  
begin  
   Top := Top + 1;  
   Content (Top) := E;  
end Push;

function Peek return Element is (Content (Top));
function Is_Full return Boolean is (Top >= Max);
function Is_Empty return Boolean is (Top = 0);
end Stack;

4.10.7 Example #7

In this version of our abstract stack unit, a copy of the stack is returned by function Get_Stack, which we call in the postcondition of Push to specify that the stack shouldn't be modified if it's full. We also assert that after we push an element on the stack, either the stack is unchanged (if it was already full) or its top element is equal to the element just pushed.

Listing 41: stack.ads
function Get_Stack return Stack_Model;

procedure Push (E : Element) with
  Post => not Is_Empty and
  (if Is_Full'Old then Get_Stack = Get_Stack'Old else Peek = E);

private

  type Stack_Model is record
    Top : Length_Type := 0;
    Content : Element_Array (1 .. Max) := (others => 0);
  end record;

end Stack;

package body Stack with
  Refined_State => (The_Stack => (Top, Content))
is
  Top : Length_Type := 0;
  Content : Element_Array (1 .. Max) := (others => 0);

procedure Push (E : Element) is
begin
  if Top <= Max then
    return;
  end if;
  Top := Top + 1;
  Content (Top) := E;
end Push;

function Peek return Element is (Content (Top));
function Is_Full return Boolean is (Top >= Max);
function Is_Empty return Boolean is (Top = 0);

function Get_Stack return Stack_Model is (Stack_Model'(Top, Content));
end Stack;

with Stack; use Stack;

procedure Use_Stack (E : Element) with
  Pre => not Is_Empty
is
  F : Element := Peek;
begin
  Push (E);
  pragma Assert (Peek = E or Peek = F);
end Use_Stack;

Code block metadata

Project: Courses.Intro_To_Spark.State_Abstraction.Example_07
MD5: 4831aa7f018f2e2d4e6d102095f8f631

Prover output
Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
use_stack.adb:9:19: medium: assertion might fail [possible fix: precondition of ...
subprogram at line 3 should mention E]
gnatprove: unproved check messages considered as errors

This program is correct, but GNATprove can't prove the assertion in Use_Stack. Indeed, even if Get_Stack is an expression function, its body isn't visible outside of Stack's body, where it's defined.

4.10.8 Example #8

Let's move the definition of Get_Stack and other expression functions inside the private part of the spec of Stack.

Listing 44: stack.ads

```
package Stack with
  Abstract_State => The_Stack
is
  pragma Unevaluated_Use_Of_Old (Allow);

  type Stack_Model is private;
  type Element is new Integer;
  type Element_Array is array (Positive range <>) of Element;
  Max : constant Natural := 100;
  subtype Length_Type is Natural range 0 .. Max;

  function Peek return Element with Pre => not Is_Empty;
  function Is_Full return Boolean;
  function Is_Empty return Boolean;
  function Get_Stack return Stack_Model;

  procedure Push (E : Element) with
    Post => not Is_Empty and
    (if Is_Full'Old then Get_Stack = Get_Stack'Old else Peek = E);
private

  Top : Length_Type := 0 with Part_Of => The_Stack;
  Content : Element_Array (1 .. Max) := (others => 0) with
    Part_Of => The_Stack;

  type Stack_Model is record
    Top : Length_Type := 0;
    Content : Element_Array (1 .. Max) := (others => 0);
  end record;

  function Peek return Element is (Content (Top));
  function Is_Full return Boolean is (Top >= Max);
  function Is_Empty return Boolean is (Top = 0);

  function Get_Stack return Stack_Model is (Stack_Model'(Top, Content));
end Stack;
```
Introduction to SPARK

Listing 45: stack.adb

package body Stack with
  Refined_State => (The_Stack => (Top, Content))

is

procedure Push (E : Element) is
begin
  if Top >= Max then
    return;
  end if;
  Top := Top + 1;
  Content (Top) := E;
end Push;
end Stack;

Listing 46: use_stack.adb

with Stack; use Stack;

procedure Use_Stack (E : Element) with
  Pre => not Is_Empty
is
  F : Element := Peek;
begin
  Push (E);
  pragma Assert (Peek = E or Peek = F);
end Use_Stack;

Code block metadata

Project: Courses.Intro_To_Spark.State_Abstraction.Example_08
MD5: 7e5204d3f69e71c212e7263906a89da4

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
use_stack.adb:6:19: info: precondition proved
use_stack.adb:9:19: info: precondition proved
use_stack.adb:9:19: info: assertion proved
use_stack.adb:9:31: info: precondition proved
stack.adb:10:30: info: range check proved
stack.adb:11:16: info: index check proved
stack.ads:19:14: info: precondition proved
stack.ads:20:60: info: precondition proved
stack.ads:33:55: info: index check proved

This example is correct. GNATprove can verify the assertion in Use_Stack because it has visibility to Get_Stack's body.

4.10. Code Examples / Pitfalls 97
4.10.9 Example #9

Package Data defines three variables, Data_1, Data_2 and Data_3, that are initialized at elaboration (in Data's package body) from an external interface that reads the file system.

Listing 47: external_interface.ads

```ada
package External_Interface with
  Abstract_State => File_System,
  Initializes => File_System
is
  type Data_Type_1 is new Integer;
  type Data_Type_2 is new Integer;
  type Data_Type_3 is new Integer;

  type Data_Record is record
    Field_1 : Data_Type_1;
    Field_2 : Data_Type_2;
    Field_3 : Data_Type_3;
  end record;

  procedure Read_Data (File_Name : String; Data : out Data_Record)
    with Global => File_System;
end External_Interface;
```

Listing 48: data.ads

```ada
with External_Interface; use External_Interface;

package Data with
  Initializes => (Data_1, Data_2, Data_3)
is
  pragma Elaborate_Body;

  Data_1 : Data_Type_1;
  Data_2 : Data_Type_2;
  Data_3 : Data_Type_3;
end Data;
```

Listing 49: data.adb

```ada
with External_Interface;
pragma Elaborate_All (External_Interface);

package body Data is
begin
  declare
    Data_Read : Data_Record;
  begin
    Read_Data ("data_file_name", Data_Read);
    Data_1 := Data_Read.Field_1;
    Data_2 := Data_Read.Field_2;
    Data_3 := Data_Read.Field_3;
  end;
end Data;
```

Code block metadata

Project: Courses.Intro_To_Spark.State_Abstraction.Example_09
MD5: 0ca44501f0c991865ea50d2ef663d992
Introduction to SPARK

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
data.adb:9:07: high: "External_Interface.File_System" must be mentioned as an input of the Initializes aspect of "Data" (SPARK RM 7.1.5.11)
gnatprove: unproved check messages considered as errors

This example isn't correct. The dependency between Data_1's, Data_2's, and Data_3's initial values and File_System must be listed in Data's Initializes aspect.

4.10.10 Example #10

Let's remove the Initializes contract on package Data.

Listing 50: external_interface.ads

```pascal
package External_Interface with
  Abstract_State => File_System,
  Initializes    => File_System
is
  type Data_Type_1 is new Integer;
  type Data_Type_2 is new Integer;
  type Data_Type_3 is new Integer;

  type Data_Record is record
    Field_1 : Data_Type_1;
    Field_2 : Data_Type_2;
    Field_3 : Data_Type_3;
  end record;

  procedure Read_Data (File_Name : String; Data : out Data_Record)
  with Global => File_System;
end External_Interface;
```

Listing 51: data.ads

```pascal
with External_Interface; use External_Interface;

package Data is
  pragma Elaborate Body;

  Data_1 : Data_Type_1;
  Data_2 : Data_Type_2;
  Data_3 : Data_Type_3;
end Data;
```

Listing 52: data.adb

```pascal
with External_Interface;
pragma Elaborate All (External_Interface);

package body Data is
begin
  declare
    Data_Read : Data_Record;
  begin
    Read_Data ("data_file_name", Data_Read);
    Data_1 := Data_Read.Field_1;
  end;
```

(continues on next page)
Data_2 := Data_Read.Field_2;
Data_3 := Data_Read.Field_3;
end Data;

Code block metadata

Project: Courses.Intro_To_Spark.State_Abstraction.Example_10
MD5: 60cba2c920c7b1031d13c82a982ed0e9

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: analysis of data and information flow ...
data.adb:7:07: info: initialization of "Data_Read" proved
external_interface.ads:3:03: info: flow dependencies proved

This example is correct. Since Data has no Initializes aspect, GNATprove computes the set of variables initialized during its elaboration as well as their dependencies.
PROOF OF FUNCTIONAL CORRECTNESS

This section is dedicated to the functional correctness of programs. It presents advanced proof features that you may need to use for the specification and verification of your program's complex properties.

5.1 Beyond Program Integrity

When we speak about the correctness of a program or subprogram, we mean the extent to which it complies with its specification. Functional correctness is specifically concerned with properties that involve the relations between the subprogram's inputs and outputs, as opposed to other properties such as running time or memory consumption.

For functional correctness, we usually specify stronger properties than those required to just prove program integrity. When we're involved in a certification processes, we should derive these properties from the requirements of the system, but, especially in non-certification contexts, they can also come from more informal sources, such as the program's documentation, comments in its code, or test oracles.

For example, if one of our goals is to ensure that no runtime error is raised when using the result of the function \texttt{Find} below, it may be enough to know that the result is either 0 or in the range of \texttt{A}. We can express this as a postcondition of \texttt{Find}.

Listing 1: show_find.ads

```ada
package Show_Find is

  type Nat_Array is array (Positive range <>) of Natural;
  function Find (A : Nat_Array; E : Natural) return Natural with
    Post => Find'Result in 0 | A'Range;

end Show_Find;
```

Listing 2: show_find.adb

```ada
package body Show_Find is

  function Find (A : Nat_Array; E : Natural) return Natural is
    begin
      for I in A'Range loop
        if A (I) = E then
          return I;
        end if;
      end loop;
      return 0;
    end Find;

(continues on next page)```
end Show_Find;

**Code block metadata**

*Project*: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Find_1  
*MD5*: d8f4ace6620fd46af170977c29947289

**Prover output**

Phase 1 of 2: generation of Global contracts ...  
Phase 2 of 2: flow analysis and proof ...  
show_find.adb:7:20: info: range check proved  
show_find.ads:6:14: info: postcondition proved

In this case, it's automatically proved by GNATprove.

However, to be sure that Find performs the task we expect, we may want to verify more complex properties of that function. For example, we want to ensure it returns an index of A where E is stored and returns 0 only if E is nowhere in A. Again, we can express this as a postcondition of Find.

Listing 3: show_find.ads

```ada
package Show_Find is
  type Nat_Array is array (Positive range <>) of Natural;
  function Find (A : Nat_Array; E : Natural) return Natural with
    Post =>
      (if (for all I in A'Range => A (I) /= E)
        then Find'Result = 0
        else Find'Result in A'Range and then A (Find'Result) = E);
end Show_Find;
```

Listing 4: show_find.adb

```ada
package body Show_Find is
  function Find (A : Nat_Array; E : Natural) return Natural is
    begin
      for I in A'Range loop
        if A (I) = E then
          return I;
        end if;
      end loop;
      return 0;
    end Find;
end Show_Find;
```

**Code block metadata**

*Project*: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Find_2  
*MD5*: 8c12b9768228a3ea45ca02199f65057b

**Prover output**

Phase 1 of 2: generation of Global contracts ...  
Phase 2 of 2: flow analysis and proof ...

(continues on next page)
This time, GNATprove can't prove this postcondition automatically, but we'll see later that we can help GNATprove by providing a loop invariant, which is checked by GNATprove and allows it to automatically prove the postcondition for \texttt{Find}.

Writing at least part of your program's specification in the form of contracts has many advantages. You can execute those contracts during testing, which improves the maintainability of the code by detecting discrepancies between the program and its specification in earlier stages of development. If the contracts are precise enough, you can use them as oracles to decide whether a given test passed or failed. In that case, they can allow you to verify the outputs of specific subprograms while running a larger block of code. This may, in certain contexts, replace the need for you to perform unit testing, instead allowing you to run integration tests with assertions enabled. Finally, if the code is in SPARK, you can also use GNATprove to formally prove these contracts.

The advantage of a formal proof is that it verifies all possible execution paths, something which isn't always possible by running test cases. For example, during testing, the postcondition of the subprogram \texttt{Find} shown below is checked dynamically for the set of inputs for which \texttt{Find} is called in that test, but just for that set.

\begin{verbatim}
package Show_Find is
  type Nat_Array is array (Positive range <>) of Natural;
  function Find (A : Nat_Array; E : Natural) return Natural with
  Post =>
    (if (for all I in A'Range => A (I) /= E)
      then Find'Result = 0
    else Find'Result in A'Range and then A (Find'Result) = E);
end Show_Find;

package body Show_Find is
  function Find (A : Nat_Array; E : Natural) return Natural is
  begin
    for I in A'Range loop
      if A (I) = E then
        return I; end if;
    end loop;
  return 0;
end Find;
end Show_Find;

with Ada.Text_IO; use Ada.Text_IO;
with Show_Find; use Show_Find;
procedure Use_Find with
  SPARK_Mode => Off
\end{verbatim}
is
Seq : constant Nat_Array (1 .. 3) := (1, 5, 3);
Res : Natural;
begin
Res := Find (Seq, 3);
Put_Line ("Found 3 in index #" & Natural'Image (Res) & " of array");
end Use_Find;

5.2 Advanced Contracts

Contracts for functional correctness are usually more complex than contracts for program integrity, so they more often require you to use the new forms of expressions introduced by the Ada 2012 standard. In particular, quantified expressions, which allow you to specify properties that must hold for all or for at least one element of a range, come in handy when specifying properties of arrays.

As contracts become more complex, you may find it useful to introduce new abstractions to improve the readability of your contracts. Expression functions are a good means to this end because you can retain their bodies in your package’s specification.

Finally, some properties, especially those better described as invariants over data than as properties of subprograms, may be cumbersome to express as subprogram contracts. Type predicates, which must hold for every object of a given type, are usually a better match for this purpose. Here’s an example.
### 5.2.1 Ghost Code

As the properties you need to specify grow more complex, you may have entities that are only needed because they are used in specifications (contracts). You may find it important to ensure that these entities can't affect the behavior of the program or that they're completely removed from production code. This concept, having entities that are only used for specifications, is usually called having ghost code and is supported in SPARK by the Ghost aspect.

You can use Ghost aspects to annotate any entity including variables, types, subprograms, and packages. If you mark an entity as Ghost, GNATprove ensures it can't affect the program's behavior. When the program is compiled with assertions enabled, ghost code is executed like normal code so it can execute the contracts using it. You can also instruct the compiler to not generate code for ghost entities.

Consider the procedure Do_Something below, which calls a complex function on its input, \(X\), and wants to check that the initial and modified values of \(X\) are related in that complex way.

Listing 9: show_ghost.ads

```ada
package Show_Ghost is
  type T is record
    A, B, C, D, E : Boolean;
  end record;
end Show_Ghost;
```
function Formula (X : T) return Boolean is
  ((X.A and X.B) or (X.C and (X.D or X.E)));

function Is_Correct (X, Y : T) return Boolean is
  (Formula (X) = Formula (Y));

procedure Do_Something (X : in out T);
end Show_Ghost;

package body Show_Ghost is
  procedure Do_Some_Complex_Stuff (X : in out T) is
    begin
      X := T'(X.B, X.A, X.C, X.E, X.D);
    end Do_Some_Complex_Stuff;

  procedure Do_Something (X : in out T) is
    X_Init : constant T := X with Ghost;
    begin
      Do_Some_Complex_Stuff (X);
      pragma Assert (Is_Correct (X_Init, X));
      -- It is OK to use X_Init inside an assertion.
    end Do_Something;
  end Show_Ghost;

package Show_Ghost is
  type T is record
    A, B, C, D, E : Boolean;
  end record;

  function Formula (X : T) return Boolean is
    ((X.A and X.B) or (X.C and (X.D or X.E)));

  function Is_Correct (X, Y : T) return Boolean is
    (Formula (X) = Formula (Y));
end Show_Ghost;

Do_Something stores the initial value of X in a ghost constant, X_Init. We reference it in an assertion to check that the computation performed by the call to Do_Some_Complex_Stuff modified the value of X in the expected manner.

However, X_Init can't be used in normal code, for example to restore the initial value of X.
procedure Do_Something (X : in out T);

end Show_Ghost;

Listing 12: show_ghost.adb

package body Show_Ghost is

procedure Do_Some_Complex_Stuff (X : in out T) is
begin
X := T'(X.B, X.A, X.C, X.E, X.D);
end Do_Some_Complex_Stuff;
procedure Do_Something (X : in out T) is
  X_Init : constant T := X with Ghost;
begin
  Do_Some_Complex_Stuff (X);
  pragma Assert (Is_Correct (X_Init, X));
  X := X_Init; -- ERROR
end Do_Something;
end Show_Ghost;

Listing 13: use_ghost.adb

with Show_Ghost; use Show_Ghost;

procedure Use_Ghost is
  X : T := (True, True, False, False, True);
begin
  Do_Something (X);
end Use_Ghost;

MD5: 464bb4bc355a648e2b92940ec80b4717

Build output

show_ghost.adb:14:12: error: ghost entity cannot appear in this context
gprbuild: *** compilation phase failed

Prover output

Phase 1 of 2: generation of Global contracts ...
show_ghost.adb:14:12: error: ghost entity cannot appear in this context
gnatprove: error during generation of Global contracts

When compiling this example, the compiler flags the use of X_Init as illegal, but more complex cases of interference between ghost and normal code may sometimes only be detected when you run GNATprove.

5.2. Advanced Contracts 107
5.2.2 Ghost Functions

Functions used only in specifications are a common occurrence when writing contracts for functional correctness. For example, expression functions used to simplify or factor out common patterns in contracts can usually be marked as ghost.

But ghost functions can do more than improve readability. In real-world programs, it's often the case that some information necessary for functional specification isn't accessible in the package's specification because of abstraction.

Making this information available to users of the packages is generally out of the question because that breaks the abstraction. Ghost functions come in handy in that case since they provide a way to give access to that information without making it available to normal client code.

Let's look at the following example.

Listing 14: stacks.ads

```
package Stacks is
  pragma Unevaluated_Use_Of_Old (Allow);

  type Stack is private;
  type Element is new Natural;
  type Element_Array is array (Positive range <>) of Element;
  Max : constant Natural := 100;

  function Get_Model (S : Stack) return Element_Array with Ghost;
  -- Returns an array as a model of a stack.

  procedure Push (S : in out Stack; E : Element) with
  Pre => Get_Model (S)'Length < Max,
  Post => Get_Model (S) = Get_Model (S)'Old & E;

private
  subtype Length_Type is Natural range 0 .. Max;
  type Stack is record
    Top : Length_Type := 0;
    Content : Element_Array (1 .. Max) := (others => 0);
  end record;

end Stacks;
```

Here, the type Stack is private. To specify the expected behavior of the Push procedure, we need to go inside this abstraction and access the values of the elements stored in S. For this, we introduce a function Get_Model that returns an array as a representation of the stack. However, we don't want code that uses the Stack package to use Get_Model in normal code since this breaks our stack's abstraction.
Here's an example of trying to break that abstraction in the subprogram Peek below.

Listing 15: stacks.ads

```ada
package Stacks is

  pragma Unevaluated_Use_Of_Old (Allow);

  type Stack is private;

  type Element is new Natural;
  type Element_Array is array (Positive range <>) of Element;
  Max : constant Natural := 100;

  function Get_Model (S : Stack) return Element_Array with Ghost;  -- Returns an array as a model of a stack.

  procedure Push (S : in out Stack; E : Element) with
    Pre => Get_Model (S)'Length < Max,
    Post => Get_Model (S) = Get_Model (S)'Old & E;

  function Peek (S : Stack; I : Positive) return Element is
    (Get_Model (S) (I));  -- ERROR

private

  subtype Length_Type is Natural range 0 .. Max;

  type Stack is record
    Top : Length_Type := 0;
    Content : Element_Array (1 .. Max) := (others => 0);
  end record;
end Stacks;
```

We see that marking the function as `Ghost` achieves this goal: it ensures that the subprogram `Get_Model` is never used in production code.

### 5.2.3 Global Ghost Variables

Though it happens less frequently, you may have specifications requiring you to store additional information in global variables that isn't needed in normal code. You should mark these global variables as ghost, allowing the compiler to remove them when assertions aren't enabled. You can use these variables for any purpose within the contracts that make up your specifications. A common scenario is writing specifications for subprograms that modify a complex or private global data structure: you can use these variables to provide a model for that structure that's updated by the ghost code as the program modifies the data structure itself.

You can also use ghost variables to store information about previous runs of subprograms.
to specify temporal properties. In the following example, we have two procedures, one that accesses a state \( A \) and the other that accesses a state \( B \). We use the ghost variable \( \text{Last\_Accessed\_Is\_A} \) to specify that \( B \) can’t be accessed twice in a row without accessing \( A \) in between.

Listing 16: call_sequence.ads

```ada
package Call_Sequence is
  type T is new Integer;

  Last_Accessed_Is_A : Boolean := False with Ghost;

  procedure Access_A with Post => Last_Accessed_Is_A;

  procedure Access_B with Pre => Last_Accessed_Is_A,
                            Post => not Last_Accessed_Is_A;
  -- B can only be accessed after A
end Call_Sequence;
```

Listing 17: call_sequence.adb

```ada
package body Call_Sequence is

  procedure Access_A is
  begin
    -- ...
    Last_Accessed_Is_A := True;
  end Access_A;

  procedure Access_B is
  begin
    -- ...
    Last_Accessed_Is_A := False;
  end Access_B;
end Call_Sequence;
```

Listing 18: main.adb

```ada
with Call_Sequence; use Call_Sequence;

procedure Main is
begin
  Access_A;
  Access_B;
  Access_B; -- ERROR
end Main;
```

Code block metadata

MD5: f33fa2ad2bd31eb03d4400c78f22eb71

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...

(continues on next page)
Runtime output

raised ADAASSERTIONSASSERTION_ERROR : failed precondition from call_sequence.
ads:11

Let's look at another example. The specification of a subprogram's expected behavior is sometimes best expressed as a sequence of actions it must perform. You can use global ghost variables that store intermediate values of normal variables to write this sort of specification more easily.

For example, we specify the subprogram Do_Two_Things below in two steps, using the ghost variable V_Interm to store the intermediate value of V between those steps. We could also express this using an existential quantification on the variable V_Interm, but it would be impractical to iterate over all integers at runtime and this can't always be written in SPARK because quantification is restricted to for loop patterns.

Finally, supplying the value of the variable may help the prover verify the contracts.

Listing 19: action_sequence.ads

```ada
package Action_Sequence is
  type T is new Integer;
  V_Interm : T with Ghost;
  function Firstliced.Done (X, Y : T) return Boolean with Ghost;
  function Second.Done (X, Y : T) return Boolean with Ghost;
  procedure Do_Two_Things (V : in out T) with
    Post => First.Done (V'Old, V_Interm)
    and then Second.Done (V_Interm, V);
end Action_Sequence;
```

Code block metadata

MD5: 2ffbd2cb187c0a81423c78e0989d62f0

Prover output

Phase 1 of 2: generation of Global contracts ...  
Phase 2 of 2: flow analysis and proof ...

Note: For more details on ghost code, see the SPARK User's Guide24.

5.3 Guide Proof

Since properties of interest for functional correctness are more complex than those involved in proofs of program integrity, we expect GNATprove to initially be unable to verify them even though they're valid. You'll find the techniques we discussed in Debugging Failed Proof Attempts (page 56) to come in handy here. We now go beyond those techniques and focus on more ways of improving results in the cases where the property is valid but GNATprove can't prove it in a reasonable amount of time.

In those cases, you may want to try and guide GNATprove to either complete the proof or strip it down to a small number of easily-reviewable assumptions. For this purpose, you can add assertions to break complex proofs into smaller steps.

```plaintext
pragma Assert (Assertion_Checked_By_The_Tool);
-- info: assertion proved
pragma Assert (Assumption_Validated_By_Other_Means);
-- medium: assertion might fail
pragma Assume (Assumption_Validated_By_Other_Means);
-- The tool does not attempt to check this expression.
-- It is recorded as an assumption.
```

One such intermediate step you may find useful is to try to prove a theoretically-equivalent version of the desired property, but one where you've simplified things for the prover, such as by splitting up different cases or inlining the definitions of functions.

Some intermediate assertions may not be proved by GNATprove either because it's missing some information or because the amount of information available is confusing. You can verify these remaining assertions by other means such as testing (since they're executable) or by review. You can then choose to instruct GNATprove to ignore them, either by turning them into assumptions, as in our example, or by using a `pragma Annotate`. In both cases, the compiler generates code to check these assumptions at runtime when you enable assertions.

5.3.1 Local Ghost Variables

You can use ghost code to enhance what you can express inside intermediate assertions in the same way we did above to enhance our contracts in specifications. In particular, you'll commonly have local variables or constants whose only purpose is to be used in assertions. You'll mostly use these ghost variables to store previous values of variables or expressions you want to refer to in assertions. They're especially useful to refer to initial values of parameters and expressions since the 'Old attribute is only allowed in postconditions.

In the example below, we want to help GNATprove verify the postcondition of P. We do this by introducing a local ghost constant, X_Init, to represent this value and writing an assertion in both branches of an if statement that repeats the postcondition, but using X_Init.

```plaintext
Listing 20: show_local_ghost.ads

package Show_Local_Ghost is
    type T is new Natural;
    function F (X, Y : T) return Boolean is (X > Y) with Ghost;
    function Condition (X : T) return Boolean is (X mod 2 = 0);
```

(continues on next page)
You can also use local ghost variables for more complex purposes such as building a data structure that serves as witness for a complex property of a subprogram. In our example, we want to prove that the Sort procedure doesn't create new elements, that is, that all the elements present in \( A \) after the sort were in \( A \) before the sort. This property isn't enough to ensure that a call to Sort produces a value for \( A \) that's a permutation of its value before the call (or that the values are indeed sorted). However, it's already complex for a prover to verify because it involves a nesting of quantifiers. To help GNATprove, you may find it useful to store, for each index \( I \), an index \( J \) that has the expected property.

```ada
procedure Sort (A : in out Nat_Array) with
Post => (for all I in A'Range =>
  (for some J in A'Range => A (I) = A'Old (J)))
begin
  Permutation : Index_Array := (1 => 1, 2 => 2, ...) with Ghost;
  ...
end Sort;
```

5.3.2 Ghost Procedures

Ghost procedures can't affect the value of normal variables, so they're mostly used to perform operations on ghost variables or to group together a set of intermediate assertions.

Abstracting away the treatment of assertions and ghost variables inside a ghost procedure has several advantages. First, you're allowed to use these variables in any way you choose in code inside ghost procedures. This isn't the case outside ghost procedures, where the only ghost statements allowed are assignments to ghost variables and calls to ghost procedures.

As an example, the for loop contained in Increase_A couldn't appear by itself in normal code.

```
package Show_Ghost_Proc is

  type Nat_Array is array (Integer range <>) of Natural;

  A : Nat_Array (1 .. 100) with Ghost;

  procedure Increase_A with
    Ghost,
    Pre => (for all I in A'Range => A (I) < Natural'Last);

end Show_Ghost_Proc;
```

```
package body Show_Ghost_Proc is

  procedure Increase_A is
    begin
      for I in A'Range loop
        A (I) := A (I) + 1;
      end loop;
    end Increase_A;

end Show_Ghost_Proc;
```

Using the abstraction also improves readability by hiding complex code that isn't part of the functional behavior of the subprogram. Finally, it can help GNATprove by abstracting away assertions that would otherwise make its job more complex.

In the example below, calling Prove_P with X as an operand only adds P (X) to the proof context instead of the larger set of assertions required to verify it. In addition, the proof of P need only be done once and may be made easier not having any unnecessary information present in its context while verifying it. Also, if GNATprove can't fully verify Prove_P, you can review the remaining assumptions more easily since they're in a smaller context.
5.3.3 Handling of Loops

When the program involves a loop, you're almost always required to provide additional annotations to allow GNATprove to complete a proof because the verification techniques used by GNATprove don't handle cycles in a subprogram's control flow. Instead, loops are flattened by dividing them into several acyclic parts.

As an example, let's look at a simple loop with an exit condition.

```
Stmt1;
loop
  Stmt2;
  exit when Cond;
  Stmt3;
end loop;
Stmt4;
```

As shown below, the control flow is divided into three parts.

![Control Flow Diagram](image)

The first, shown in yellow, starts earlier in the subprogram and enters the loop statement. The loop itself is divided into two parts. Red represents a complete execution of the loop's body: an execution where the exit condition isn't satisfied. Blue represents the last execution of the loop, which includes some of the subprogram following it. For that path, the exit condition is assumed to hold. The red and blue parts are always executed after the yellow one.

GNATprove analyzes these parts independently since it doesn't have a way to track how variables may have been updated by an iteration of the loop. It forgets everything it knows about those variables from one part when entering another part. However, values of constants and variables that aren't modified in the loop are not an issue.

In other words, handling loops in that way makes GNATprove imprecise when verifying a subprogram involving a loop: it can't verify a property that relies on values of variables modified inside the loop. It won't forget any information it had on the value of constants or unmodified variables, but it nevertheless won't be able to deduce new information about them from the loop.

For example, consider the function `Find` which iterates over the array `A` and searches for an element where `E` is stored in `A`.

```
package Show_Find is

  procedure Find (E : T) with Ghost,
  Global => null,
  Post => P (E);

Listing 24: show_find.ads
```

(continues on next page)
type Nat_Array is array (Positive range <>) of Natural;

function Find (A : Nat_Array; E : Natural) return Natural;

end Show_Find;

Listing 25: show_find.adb

package body Show_Find is

function Find (A : Nat_Array; E : Natural) return Natural is
  begin
    for I in A'Range loop
      pragma Assert (for all J in A'First .. I - 1 => A (J) /= E);
      -- assertion is not proved
      if A (I) = E then
        return I;
      end if;
      pragma Assert (A (I) /= E);
      -- assertion is proved
    end loop;
    return 0;
  end Find;
end Show_Find;

Code block metadata

Project: Courses.Intro To_Spark.Proof of Functional Correctness.Loop
MD5: cb9cd0cb102c3baba3b21a788b6e4ae3

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
  show_find.adb:6:51: info: overflow check proved
  show_find.adb:6:58: medium: assertion might fail, cannot prove A (J) /= E,
    --[possible fix: subprogram at show_find.ads:5 should mention A and E in a
    --precondition]
  show_find.adb:6:61: info: index check proved
  show_find.adb:9:20: info: range check proved
  show_find.adb:11:25: info: assertion proved
  gnatprove: unproved check messages considered as errors

At the end of each loop iteration, GNATprove knows that the value stored at index I in A
must not be E. (If it were, the loop wouldn't have reached the end of the iteration.) This
proves the second assertion. But it's unable to aggregate this information over multiple
loop iterations to deduce that it's true for all the indexes smaller than I, so it can't prove
the first assertion.
5.3.4 Loop Invariants

To overcome these limitations, you can provide additional information to GNATprove in the form of a loop invariant. In SPARK, a loop invariant is a Boolean expression which holds true at every iteration of the loop. Like other assertions, you can have it checked at runtime by compiling the program with assertions enabled.

The major difference between loop invariants and other assertions is the way it's treated for proofs. GNATprove performs the proof of a loop invariant in two steps: first, it checks that it holds for the first iteration of the loop and then it checks that it holds in an arbitrary iteration assuming it held in the previous iteration. This is called proof by induction.

As an example, let's add a loop invariant to the Find function stating that the first element of A is not E.

Listing 26: show_find.ads

```
package Show_Find is
  type Nat_Array is array (Positive range <>) of Natural;
  function Find (A : Nat_Array; E : Natural) return Natural;
end Show_Find;
```

Listing 27: show_find.adb

```
package body Show_Find is
  function Find (A : Nat_Array; E : Natural) return Natural is begin
    for I in A'Range loop
      pragma Loop_Invariant (A (A'First) /= E);
      -- loop invariant not proved in first iteration
      -- but preservation of loop invariant is proved
      if A (I) = E then
        return I;
      end if;
    end loop;
    return 0;
  end Find;
end Show_Find;
```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Loop_Invariant_1
MD5: 8d5fefdca9deacd4eb50850be91fbe

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_find.adb:6:33: info: loop invariant preservation proved
show_find.adb:6:33: medium: loop invariant might fail in first iteration [possible fixing: subprogram at show_find.ads:5 should mention A and E in a precondition]
show_find.adb:6:37: info: index check proved
show_find.adb:10:20: info: range check proved
gnatprove: unproved check messages considered as errors

To verify this invariant, GNATprove generates two checks. The first checks that the assertion holds in the first iteration of the loop. This isn't verified by GNATprove. And indeed there's no reason to expect the first element of \( A \) to always be different from \( E \) in this iteration. However, the second check is proved: it's easy to deduce that if the first element of \( A \) was not \( E \) in a given iteration it's still not \( E \) in the next. However, if we move the invariant to the end of the loop, then it is successfully verified by GNATprove.

Not only do loop invariants allow you to verify complex properties of loops, but GNATprove also uses them to verify other properties, such as the absence of runtime errors over both the loop's body and the statements following the loop. More precisely, when verifying a runtime check or other assertion there, GNATprove assumes that the last occurrence of the loop invariant preceding the check or assertion is true.

Let's look at a version of \texttt{Find} where we use a loop invariant instead of an assertion to state that none of the array elements seen so far are equal to \( E \).

**Listing 28: show_find.ads**

```
package Show_Find is
  type Nat_Array is array (Positive range <>) of Natural;
  function Find (A : Nat_Array; E : Natural) return Natural;
end Show_Find;
```

**Listing 29: show_find.adb**

```
package body Show_Find is
  function Find (A : Nat_Array; E : Natural) return Natural is
  begin
    for I in A'Range loop
      pragma Loop_Invariant
      (for all J in A'First .. I - 1 => A (J) /= E);
      if A (I) = E then
        return I;
      end if;
    end loop;
    pragma Assert (for all I in A'Range => A (I) /= E);
    return 0;
  end Find;
end Show_Find;
```

This version is fully verified by GNATprove! This time, it proves that the loop invariant holds
in every iteration of the loop (separately proving this property for the first iteration and then for the following iterations). It also proves that none of the elements of A are equal to E after the loop exits by assuming that the loop invariant holds in the last iteration of the loop.

Note: For more details on loop invariants, see the SPARK User's Guide.

Finding a good loop invariant can turn out to be quite a challenge. To make this task easier, let's review the four good properties of a good loop invariant:

<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>INIT</td>
<td>It should be provable in the first iteration of the loop.</td>
</tr>
<tr>
<td>INSIDE</td>
<td>It should allow proving the absence of run-time errors and local assertions inside the loop.</td>
</tr>
<tr>
<td>AFTER</td>
<td>It should allow proving absence of run-time errors, local assertions, and the subprogram postcondition after the loop.</td>
</tr>
<tr>
<td>PRESERVE</td>
<td>It should be provable after the first iteration of the loop.</td>
</tr>
</tbody>
</table>

Let's look at each of these in turn. First, the loop invariant should be provable in the first iteration of the loop (INIT). If your invariant fails to achieve this property, you can debug the loop invariant's initialization like any failing proof attempt using strategies for Debugging Failed Proof Attempts (page 56).

Second, the loop invariant should be precise enough to allow GNATprove to prove absence of runtime errors in both statements from the loop's body (INSIDE) and those following the loop (AFTER). To do this, you should remember that all information concerning a variable modified in the loop that's not included in the invariant is forgotten by GNATprove. In particular, you should take care to include in your invariant what's usually called the loop's frame condition, which lists properties of variables that are true throughout the execution of the loop even though those variables are modified by the loop.

Finally, the loop invariant should be precise enough to prove that it's preserved through successive iterations of the loop (PRESERVE). This is generally the trickiest part. To understand why GNATprove hasn't been able to verify the preservation of a loop invariant you provided, you may find it useful to repeat it as local assertions throughout the loop's body to determine at which point it can no longer be proved.

As an example, let's look at a loop that iterates through an array A and applies a function F to each of its elements.

```
package Show_Map is
  type Nat_Array is array (Positive range <>) of Natural;
  function F (V : Natural) return Natural is
    (if V /= Natural'Last then V + 1 else V);
  procedure Map (A : in out Nat_Array);
end Show_Map;
```

Listing 30: show_map.ads

package body Show_Map is

   procedure Map (A : in out Nat_Array) is
      A_I : constant Nat_Array := A with Ghost;
   begin
      for K in A'Range loop
         A (K) := F (A (K));
      pragma Loop Invariant
         (for all J in A'First .. K => A (J) = F (A'Loop_Entry (J)));
      end loop;
      pragma Assert (for all K in A'Range => A (K) = F (A_I (K)));
   end Map;
end Show_Map;

Code block metadata

MD5: 1a4583c9b2b772f79bcf29c9f0ca96a

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_map.adb:9:13: info: loop invariant initialization proved
show_map.adb:9:45: info: index check proved
show_map.adb:9:67: info: index check proved
show_map.adb:11:22: info: assertion proved
show_map.adb:11:49: info: index check proved
show_map.adb:11:62: info: index check proved
show_map.ads:6:35: info: overflow check proved

After the loop, each element of A should be the result of applying F to its previous value. We want to prove this. To specify this property, we copy the value of A before the loop into a ghost variable, A_I. Our loop invariant states that the element at each index less than K has been modified in the expected way. We use the Loop_Entry attribute to refer to the value of A on entry of the loop instead of using A_I.

Does our loop invariant have the four properties of a good loop-invariant? When launching GNATprove, we see that INIT is fulfilled: the invariant's initialization is proved. So are INSIDE and AFTER: no potential runtime errors are reported and the assertion following the loop is successfully verified.

The situation is slightly more complex for the PRESERVE property. GNATprove manages to prove that the invariant holds after the first iteration thanks to the automatic generation of frame conditions. It was able to do this because it completes the provided loop invariant with the following frame condition stating what part of the array hasn't been modified so far:

   pragma Loop Invariant
      (for all J in K .. A'Last => A (J) = (if J > K then A'Loop_Entry (J)));

GNATprove then uses both our and the internally-generated loop invariants to prove PRESERVE. However, in more complex cases, the heuristics used by GNATprove to generate the frame condition may not be sufficient and you'll have to provide one as a loop invariant. For example, consider a version of Map where the result of applying F to an element at index K is stored at index K - 1:
Introduction to SPARK

Listing 32: show_map.ads

```ada
package Show_Map is

  type Nat_Array is array (Positive range <>) of Natural;

  function F (V : Natural) return Natural is
    (if V /= Natural'Last then V + 1 else V);

  procedure Map (A : in out Nat_Array);

end Show_Map;
```

Listing 33: show_map.adb

```ada
package body Show_Map is

  procedure Map (A : in out Nat_Array) is
    A_I : constant Nat_Array := A with Ghost;
    begin
      for K in A'Range loop
        if K /= A'First then
          A (K - 1) := F (A (K));
        end if;
        pragma Loop Invariant
          (for all J in A'First .. K =>
            (if J /= A'First then A (J - 1) = F (A'Loop_Entry (J)))
            pragma Loop Invariant
            (for all J in K .. A'Last => A (J) = A'Loop_Entry (J)));
      end loop;
      pragma Assert (for all K in A'Range =>
        (if K /= A'First then A (K - 1) = F (A_I (K)))
        pragma Assert
        (for all J in A'Range =>
          (if K /= A'First then A (K - 1) = F (A_I (K)));
      end Map;

end Show_Map;
```

Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Loop_Invariant_4
MD5: 6c51768547d3baa2c19d0e33959388fe

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
show_map.adb:8:18: info: overflow check proved
show_map.adb:8:18: info: index check proved
show_map.adb:11:13: info: loop invariant initialization proved
show_map.adb:12:36: medium: loop invariant might not be preserved by an arbitrary iteration, cannot prove A (J - 1) = F (A'Loop_Entry (J))
show_map.adb:12:41: info: overflow check proved
show_map.adb:12:41: info: index check proved
show_map.adb:12:65: info: index check proved
show_map.adb:16:22: info: assertion proved
show_map.adb:17:50: info: overflow check proved
show_map.adb:17:50: info: index check proved
show_map.adb:17:65: info: index check proved
show_map.ads:6:35: info: overflow check proved
gnatprove: unproved check messages considered as errors

You need to uncomment the second loop invariant containing the frame condition in order to prove the assertion after the loop.

5.3. Guide Proof
5.4 Code Examples / Pitfalls

This section contains some code examples and pitfalls.

5.4.1 Example #1

We implement a ring buffer inside an array Content, where the contents of a ring buffer of length Length are obtained by starting at index First and possibly wrapping around the end of the buffer. We use a ghost function Get_Model to return the contents of the ring buffer for use in contracts.

```
package Ring_Buffer is

  Max_Size : constant := 100;

  type Nat_Array is array (Positive range <>) of Natural;

  function Get_Model return Nat_Array with Ghost;

  procedure Push_Last (E : Natural) with
    Pre => Get_Model'Length < Max_Size,
    Post => Get_Model'Length = Get_Model'Old'Length + 1;

end Ring_Buffer;
```

```
package body Ring_Buffer is

  subtype Length_Range is Natural range 0 .. Max_Size;
  subtype Index_Range is Natural range 1 .. Max_Size;

  Content : Nat_Array (1 .. Max_Size) := (others => 0);
  First : Index_Range := 1;
  Length : Length_Range := 0;

  function Get_Model return Nat_Array with
    Refined_Post => Get_Model'Result'Length = Length
  is
    Size : constant Length_Range := Length;
    Result : Nat_Array (1 .. Size) := (others => 0);
    begin
      if First + Length - 1 <= Max_Size then
        Result := Content (First .. First + Length - 1);
      else
        declare
          Len : constant Length_Range := Max_Size - First + 1;
        begin
          Result (1 .. Len) := Content (First .. Max_Size);
```

Note: For more details on how to write a loop invariant, see the SPARK User’s Guide\(^\text{27}\).

Result (Len + 1 .. Length) := Content (1 .. Length - Len);

end Get_Model;

procedure Push_Last (E : Natural) is
begin
  if First + Length <= Max_Size then
    Content (First + Length) := E;
  else
    Content (Length - Max_Size + First) := E;
  end if;
  Length := Length + 1;
end Push_Last;

end Ring_Buffer;

This is correct: Get_Model is used only in contracts. Calls to Get_Model make copies of the buffer's contents, which isn't efficient, but is fine because Get_Model is only used for verification, not in production code. We enforce this by making it a ghost function. We'll produce the final production code with appropriate compiler switches (i.e., not using -gnata) that ensure assertions are ignored.
**5.4.2 Example #2**

Instead of using a ghost function, Get_Model, to retrieve the contents of the ring buffer, we're now using a global ghost variable, Model.

Listing 36: ring_buffer.ads

```ada
package Ring_Buffer is

   Max_Size : constant := 100;
   subtype Length_Range is Natural range 0 .. Max_Size;
   subtype Index_Range is Natural range 1 .. Max_Size;

   type Nat_Array is array (Positive range <>) of Natural;

   type Model_Type (Length : Length_Range := 0) is record
      Content : Nat_Array (1 .. Length);
      end record
      with Ghost;
   end record

   Model : Model_Type with Ghost;

   function Valid_Model return Boolean;

   procedure Push_Last (E : Natural) with
      Pre => Valid_Model
      and then Model.Length < Max_Size,
      Post => Model.Length = Model.Length'Old + 1;

end Ring_Buffer;
```

Listing 37: ring_buffer.adb

```ada
package body Ring_Buffer is

   Content : Nat_Array (1 .. Max_Size) := (others => 0);
   First : Index_Range := 1;
   Length : Length_Range := 0;

   function Valid_Model return Boolean is
      (Model.Content'Length = Length);

   procedure Push_Last (E : Natural) is
      begin
         if First + Length <= Max_Size then
            Content (First + Length) := E;
         else
            Content (Length - Max_Size + First) := E;
         end if;
         Length := Length + 1;
      end Push_Last;

end Ring_Buffer;
```
Prover output

Phase 1 of 2: generation of Global contracts ...
ring_buffer.adb:8:08: error: ghost entity cannot appear in this context
gnatprove: error during generation of Global contracts

This example isn't correct. Model, which is a ghost variable, must not influence the return value of the normal function Valid_Model. Since Valid_Model is only used in specifications, we should have marked it as Ghost. Another problem is that Model needs to be updated inside Push_Last to reflect the changes to the ring buffer.

5.4.3 Example #3

Let's mark Valid_Model as Ghost and update Model inside Push_Last.

Listing 38: ring_buffer.ads

```ada
package Ring_Buffer is
  Max_Size : constant := 100;
  subtype Length_Range is Natural range 0 .. Max_Size;
  subtype Index_Range is Natural range 1 .. Max_Size;

  type Nat_Array is array (Positive range <>) of Natural;

  type Model_Type (Length : Length_Range := 0) is record
    Content : Nat_Array (1 .. Length);
    end record
    with Ghost;

  Model : Model_Type with Ghost;

  function Valid_Model return Boolean with Ghost;

  procedure Push_Last (E : Natural) with
    Pre => Valid_Model
    and then Model.Length < Max_Size,
    Post => Model.Length = Model.Length'Old + 1;
end Ring_Buffer;
```

Listing 39: ring_buffer.adb

```ada
package body Ring_Buffer is

  Content : Nat_Array (1 .. Max_Size) := (others => 0);
  First : Index_Range := 1;
  Length : Length_Range := 0;

  function Valid_Model return Boolean is
    (Model.Content'Length = Length);

  procedure Push_Last (E : Natural) is
    begin
      if First + Length <= Max_Size then
        Content (First + Length) := E;
```

(continues on next page)
5.4.4 Example #4

We’re now modifying Push_Last to share the computation of the new length between the operational and ghost code.

Listing 40: ring_buffer.ads

```ada
package Ring_Buffer is

   Max_Size : constant := 100;
   subtype Length_Range is Natural range 0 .. Max_Size;
   subtype Index_Range is Natural range 1 .. Max_Size;

   type Nat_Array is array (Positive range <>) of Natural;

type Model_Type (Length : Length_Range := 0) is record
   Content : Nat_Array (1 .. Length);
   end record
   with Ghost;

   Model : Model_Type with Ghost;

   function Valid_Model return Boolean with Ghost;

   procedure Push_Last (E : Natural) with
```

(continues on next page)
19 Pre => Valid_Model
   and then Model.Length < Max_Size,
20 Post => Model.Length = Model.Length'Old + 1;
end Ring_Buffer;

Listing 41: ring_buffer.adb

package body Ring_Buffer is
   Content : Nat_Array (1 .. Max_Size) := (others => 0);
   First   : Index_Range     := 1;
   Length  : Length_Range    := 0;

   function Valid_Model return Boolean is
      (Model.Content'Length = Length);
   procedure Push_Last (E : Natural) is
      New_Length : constant Length_Range := Model.Length + 1;
      begin
         if First + Length <= Max_Size then
            Content (First + Length) := E;
         else
            Content (Length - Max_Size + First) := E;
         end if;
      Length := New_Length;
      Model := (Length => New_Length,
                  Content => Model.Content & E);
   end Push_Last;
end Ring_Buffer;

Code block metadata

MD5: e27f0b4729be72d83f2cb981b1d00412

Build output

ring_buffer.adb:11:45: error: ghost entity cannot appear in this context
gprbuild: *** compilation phase failed

Prover output

Phase 1 of 2: generation of Global contracts ...
ring_buffer.adb:11:45: error: ghost entity cannot appear in this context
gnatprove: error during generation of Global contracts

This example isn’t correct. We didn’t mark local constant New_Length as Ghost, so it can’t be computed from the value of ghost variable Model. If we made New_Length a ghost constant, the compiler would report the problem on the assignment from New_Length to Length. The correct solution here is to compute New_Length from the value of the non-ghost variable Length.
5.4.5 Example #5

Let’s move the code updating Model inside a local ghost procedure, Update_Model, but still using a local variable, New_Length, to compute the length.

Listing 42: ring_buffer.ads

```ada
package Ring_Buffer is

Max_Size : constant := 100;
subtype Length_Range is Natural range 0 .. Max_Size;
subtype Index_Range is Natural range 1 .. Max_Size;

type Nat_Array is array (Positive range <>) of Natural;
type Model_Type (Length : Length_Range := 0) is record
  Content : Nat_Array (1 .. Length);
  with Ghost;
end record

Model : Model_Type with Ghost;

function Valid_Model return Boolean with Ghost;

procedure Push_Last (E : Natural) with
  Pre => Valid_Model
  and then Model.Length < Max_Size,
  Post => Model.Length = Model.Length'Old + 1;

end Ring_Buffer;
```

Listing 43: ring_buffer.adb

```ada
package body Ring_Buffer is

Content : Nat_Array (1 .. Max_Size) := (others => 0);
First : Index_Range := 1;
Length : Length_Range := 0;

function Valid_Model return Boolean is
  (Model.Content'Length = Length);

procedure Push_Last (E : Natural) is

  procedure Update_Model with Ghost is
    New_Length : constant Length_Range := Model.Length + 1;
    begin
      Model := (Length => New_Length,
                Content => Model.Content & E);
    end Update_Model;

  begin
    if First + Length <= Max_Size then
      Content (First + Length) := E;
    else
      Content (Length - Max_Size + First) := E;
    end if;
    Length := Length + 1;
    Update_Model;
  end Push_Last;

end Ring_Buffer;
```
5.4.6 Example #6

The function Max_Array takes two arrays of the same length (but not necessarily with the same bounds) as arguments and returns an array with each entry being the maximum values of both arguments at that index.

```
package Array_Util is

  type Nat_Array is array (Positive range <>) of Natural;

  function Max_Array (A, B : Nat_Array) return Nat_Array with
     Pre => A'Length = B'Length;

end Array_Util;
```

(continues on next page)
13 | J := J + 1;  
14 | end loop;  
15 | return R;  
16 | end Max_Array;  
17 | end Array_Util;  

### Code block metadata

Project: Courses.Intro_To_Spark.Proof_of_Functional_Correctness.Example_06  
MD5: 4b8a6a9b1a3d4d228fe1e944914084fe

### Prover output

Phase 1 of 2: generation of Global contracts ...  
Phase 2 of 2: flow analysis and proof ...  
array_util.adb:8:24: medium: array index check might fail [reason for check: value must be a valid index into the array] [possible fix: loop at line 7 should mention J in a loop invariant]  
array_util.adb:13:17: medium: overflow check might fail, cannot prove upper bound [reason for check: result of addition must fit in a 32-bits machine integer] [possible fix: loop at line 7 should mention J in a loop invariant]  
gnatprove: unproved check messages considered as errors

This program is correct, but GNATprove can't prove that J is always in the index range of B (the unproved index check) or even that it's always within the bounds of its type (the unproved overflow check). Indeed, when checking the body of the loop, GNATprove forgets everything about the current value of J because it's been modified by previous loop iterations. To get more precise results, we need to provide a loop invariant.

#### 5.4.7 Example #7

Let's add a loop invariant that states that J stays in the index range of B and let's protect the increment to J by checking that it's not already the maximal integer value.

### Listing 46: array_util.ads

```ada
package Array_Util is  
  type Nat_Array is array (Positive range <>) of Natural;  
  function Max_Array (A, B : Nat_Array) return Nat_Array with  
    Pre => A'Length = B'Length;  
end Array_Util;
```

### Listing 47: array_util.adb

```ada
package body Array_Util is  
  function Max_Array (A, B : Nat_Array) return Nat_Array is  
    R : Nat_Array (A'Range);  
    J : Integer := B'First;  
  begin  
    for I in A'Range loop  
      pragma Loop Invariant (J in B'Range);  
      if A (I) > B (J) then  
        R (I) := A (I);  
      end if;  
    end loop;  
  return R;  
end Max_Array;  
```

(continues on next page)
5.4.8 Example #8

We now consider a version of Max_Array which takes arguments that have the same bounds. We want to prove that Max_Array returns an array of the maximum values of both its arguments at each index.

```haskell
package Array_Util is

  type Nat_Array is array (Positive range <>) of Natural;

  function Max_Array (A, B : Nat_Array) return Nat_Array with
  Pre => A'First = B'First and A'Last = B'Last,
  Post => (for all K in A'Range =>
          Max_Array'Result (K) = Natural'Max (A (K), B (K)));

end Array_Util;
```

We now consider a version of Max_Array which takes arguments that have the same bounds. We want to prove that Max_Array returns an array of the maximum values of both its arguments at each index.

```haskell
package body Array_Util is

  function Max_Array (A, B : Nat_Array) return Nat_Array is
  begin
    R : Nat_Array (A'Range) := (others => 0);
    for I in A'Range loop
      if I < Integer'Last then
        R (I) := B (J);
      end if;
      if J < Integer'Last then
        J := J + 1;
      end if;
    end loop;
    return R;
  end Max_Array;

end Array_Util;
```

The loop invariant now allows verifying that no runtime error can occur in the loop's body (property INSIDE seen in section Loop Invariants (page 117)). Unfortunately, GNATprove fails to verify that the invariant stays valid after the first iteration of the loop (property PRESERVE). Indeed, knowing that J is in B'Range in a given iteration isn't enough to prove it'll remain so in the next iteration. We need a more precise invariant, linking J to the value of the loop index I, like J = I - A'First + B'First.
for I in A’Range loop
    pragma Loop_Invariant (for all K in A’First .. I =>
        R (K) = Natural ‘Max (A (K), B (K)));
    if A (I) > B (I) then
        R (I) := A (I);
    else
        R (I) := B (I);
    end if;
end loop;
return R;
end Max_Array;

Listing 50: main.adb

with Array_Util; use Array_Util;

procedure Main is
    A : Nat_Array := (1, 1, 2);
    B : Nat_Array := (2, 1, 0);
    R : Nat_Array (1 .. 3);
begin
    R := Max_Array (A, B);
end Main;

Code block metadata

MD5: d0a04c214a632466a4fe4ec6cb7f8842

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
main.adb:8:09: medium: length check might fail [reason for check: array must be of the appropriate length]
array_util.adb:8:35: medium: loop invariant might not be preserved by an arbitrary iteration, cannot prove R (K) = Natural ‘max
array_util.adb:8:35: medium: loop invariant might fail in first iteration, cannot prove R (K) = Natural ‘max
gnatprove: unproved check messages considered as errors

Runtime output

raised ADA ASSERTIONS ASSERTION_ERROR : Loop_Invariant failed at array_util.adb:7

Here, GNATprove doesn't manage to prove the loop invariant even for the first loop iteration (property INIT seen in section Loop Invariants (page 117)). In fact, the loop invariant is incorrect, as you can see by executing the function Max_Array with assertions enabled: at each loop iteration, R contains the maximum of A and B only until I - 1 because the I'th index wasn't yet handled.
5.4.9 Example #9

We now consider a procedural version of Max_Array which updates its first argument instead of returning a new array. We want to prove that Max_Array sets the maximum values of both its arguments into each index in its first argument.

Listing 51: array_util.ads

```
package Array_Util is

  type Nat_Array is array (Positive range <>) of Natural;

  procedure Max_Array (A : in out Nat_Array; B : Nat_Array) with
    Pre => A'First = B'First and A'Last = B'Last,
    Post => (for all K in A'Range =>
      A (K) = Natural'Max (A'Old (K), B (K)));

end Array_Util;
```

Listing 52: array_util.adb

```
package body Array_Util is

  procedure Max_Array (A : in out Nat_Array; B : Nat_Array) is
    begin
      for I in A'Range loop
        pragma LoopInvariant
        (for all K in A'First .. I - 1 =>
         A (K) = Natural'Max (A'Loop_Entry (K), B (K)));
        pragma LoopInvariant
        (for all K in I .. A'Last => A (K) = A'Loop_Entry (K));
        if A (I) <= B (I) then
          A (I) := B (I);
        end if;
      end loop;
    end Max_Array;

end Array_Util;
```
Everything is proved. The first loop invariant states that the values of \( A \) before the loop index contains the maximum values of the arguments of \( \text{Max} \_\text{Array} \) (referring to the input value of \( A \) with \( \text{A'}\text{Loop} \_\text{Entry} \)). The second loop invariant states that the values of \( A \) beyond and including the loop index are the same as they were on entry. This is the frame condition of the loop.

### 5.4.10 Example #10

Let's remove the frame condition from the previous example.

Listing 53: array_util.ads

```ada
package Array_Util is

  type Nat_Array is array (Positive range <>) of Natural;

  procedure Max_Array (A : in out Nat_Array; B : Nat_Array) with
    Pre => A'First = B'First and A'Last = B'Last,
    Post => (for all K in A'Range =>
      A (K) = Natural'Max (A'Old (K), B (K)));

end Array_Util;
```

Listing 54: array_util.adb

```ada
package body Array_Util is

  procedure Max_Array (A : in out Nat_Array; B : Nat_Array) is
    begin
      for I in A'Range loop
        pragma Loop Invariant
          (for all K in A'First .. I - 1 =>
            A (K) = Natural'Max (A'Loop_Entry (K), B (K)));
        if A (I) <= B (I) then
          A (I) := B (I);
        end if;
      end loop;

end Max_Array;
```

Code block metadata

MD5: 8bdc8432cbb3f26f58f63457408c7172

Prover output

Phase 1 of 2: generation of Global contracts ...
Phase 2 of 2: flow analysis and proof ...
array_util.adb:7:13: info: loop invariant initialization proved
array_util.adb:7:13: info: loop invariant preservation proved
array_util.adb:7:39: info: overflow check proved
array_util.adb:8:18: info: index check proved
array_util.adb:8:50: info: index check proved

(continues on next page)
Everything is still proved. GNATprove internally generates the frame condition for the loop, so it's sufficient here to state that A before the loop index contains the maximum values of the arguments of Max_Array.